LEARNING WITH UNDERSTANDING

The conceptual basis for our work at the National Center for Improving Student Learning and Achievement (NCISLA) is centered on learning with understanding. It is difficult to define understanding without engaging in circular argument. Because virtually all complex ideas or processes can be understood at a number of levels and in quite different ways, we characterize understanding as emerging or developing. As a consequence, we choose to define understanding in terms of mental activity that contributes to the development of understanding rather than as a static attribute of an individual's knowledge.

We propose five forms of mental activity from which mathematical and scientific understanding emerges: (a) constructing relationships, (b) extending and applying mathematical and scientific knowledge, (c) reflecting about experiences, (d) articulating what one knows, and (e) making mathematical and scientific knowledge one's own. These ideas are elaborated in more detail in Carpenter and Lehrer (1999), Fennema and Romberg (1999) and specific examples of how they are instantiated in classrooms are described throughout our web site.

Our conceptual analysis of professional development is based on the same fundamental conceptions of learning with understanding that we apply to students' learning of mathematics and science. In particular, professional development is designed to help teachers connect mathematical and scientific knowledge, knowledge of students' mathematical and scientific thinking, and teachers' instructional practices. As with instruction, much of the story rests with the details of learning to teach specific mathematics and science content.

Constructing Relationships

For students learning science or mathematics, new ideas take on meaning by the ways they are related to other ideas. People construct meaning for a new idea or process by relating it to ideas or processes that they already understand. Although learning with understanding entails forging connections between what the students already know and the knowledge they are learning, it is not sufficient to think of developing understanding simply as appending new concepts and processes to existing knowledge. Over the long run, developing understanding involves more than simply connecting new knowledge to prior knowledge; it also involves the creation of rich, integrated knowledge structures.

Center researchers have worked to identify learning trajectories of important mathematical and scientific ideas so that teachers can have the knowledge to help students at different points along the trajectory connect the knowledge that they already have learned to the new content they are studying and to connect less sophisticated strategies to more sophisticated processes. For example, research on the development of children's algebraic reasoning has portrayed the learning trajectory for forms of justification that children use to validate mathematical arguments, (Carpenter & Levi, 1999) and research on statistics in the middle grades has documented the trajectory of students' learning of ideas of sampling distribution and the means by which that learning was supported by instruction (Cobb, 1999; Cobb, McClain, & Gravemeijer, 2000).

Extending and Applying Mathematical and Scientific Knowledge

Perhaps the most important feature of learning with understanding is that it is generative. When students or teachers acquire knowledge with understanding, they can apply the knowledge to learn new topics and solve new and unfamiliar problems. When students or teachers do not understand, they perceive that each topic is an isolated skill. They cannot apply their skills to solve problems not explicitly taught to them, nor extend their learning to new topics.

One of the defining characteristics of learning with understanding is that knowledge is learned in ways that clarify how it can be used. When knowledge is structured as it is when it is learned with understanding, new knowledge can be related to and incorporated into existing networks of knowledge rather than creating connections on an element-by-element basis. When students and teachers see a number of critical relationships among concepts and processes, they are more likely to recognize how their existing knowledge might be related to new situations.

A fundamental assumption of our work is that for learning to be generative, students and teachers must be engaged in learning that involves the same generative processes that we expect them to apply to use their knowledge to learn new ideas and solve unfamiliar problems in the future. The work of the Center science projects illustrates how instruction engages students in generative activity. Students in these classes are regularly involved in constructing, testing, and revising models (Cartier & Stewart, 2000; Lehrer & Schauble, 1998, in press; Rosebery & Warren, 1998), and students in both science and mathematics classes formulate and evaluate conjectures (Carpenter & Levi, 1999; Cobb & McClain, in press; Kaput & Blanton, 1999). In the same way, teachers in the professional development programs extend their knowledge as they engage in formulating and testing hypotheses about student thinking and the instructional practices that influence the development of that thinking (Franke, Carpenter, Levi & Fennema, in press).

Reflection

Reflection involves the conscious examination of one’s own actions or thoughts. The routine application of skills requires little reflection as one just follows a set of familiar procedures, but reflection plays an important role in solving unfamiliar problems, whether they be mathematical or scientific problems or pedagogical problems. Problem solving often involves consciously examining the relation between one’s existing knowledge and the conditions of a problem situation. Students stand a better chance of acquiring this ability if reflection is a part of the knowledge acquisition process. To be reflective in their learning means that students and teachers consciously examine the knowledge they are acquiring, in particular how it is related to what they already know and to other knowledge they are acquiring.

Articulation

The ability to communicate or articulate one's ideas is an important goal of instruction, and it also is a benchmark of understanding. Articulation involves the communication of one's knowledge, either verbally, in writing, or through some other means like pictures, diagrams, or models. Articulation requires reflection in that it involves lifting out the critical ideas of an activity so that the essence of the activity can be communicated. In the process, the activity becomes an object of thought. In other words, in order to articulate one's ideas, it is necessary to reflect on them so that critical elements can be identified and described. Articulation, thus, requires reflection and can be thought of as a public form of reflection.

As with reflection, students and teachers initially have difficulty articulating their ideas about an unfamiliar topic or task. It is through struggling to articulate their ideas, especially in the context of mathematical and scientific symbols or models, that students develop the ability to reflect on and articulate their thinking.

Making Knowledge One's Own

Understanding involves the construction of knowledge by individuals through their own activity, so that they develop a personal investment in building knowledge. They cannot perceive their knowledge simply as something that someone else has told them or explained to them. They need to adopt a stance that knowledge is evolving and provisional: They will not view knowledge in this way if they see it as someone else's knowledge that they simply assimilate through listening, watching, and practicing.

This does not mean that students cannot learn by listening to teachers or to other students, but they have to adapt what they hear to their own ends, not simply accept what they hear because it is clearly articulated by an authority figure. This also does not mean that understanding is entirely private. The development of students' personal involvement in learning with understanding is tied to classroom practices in which communication and negotiation of meanings are important facets.

An overarching goal of our instruction is that students and teachers develop a predisposition to understand and that they strive to understand because understanding becomes important to them. This means that students and teachers themselves become reflective about the activities they engage in while learning or solving problems. They look for relationships that might give meaning to a new idea and critically examine their existing knowledge looking for new and more productive relationships. They view learning as problem solving in which the goal is to extend their knowledge.

Understanding as a Community Activity

Learning with understanding generally is thought of in terms of knowledge of individuals and much of the foregoing discussion has focused on individual learning. Learning, however, often takes place in groups, and one of the benefits of thinking about understanding as emerging rather than static is that the components of our analysis of learning with understanding can be applied to communities of learners as well as to individuals. In instruction based on Center research, class activity often involves sharing strategies and ideas with the goal of developing within the class connections among the different strategies and ideas available to it. The classes also are engaged in practices of generating knowledge. Conjectures are proposed, and the members of the class often work together to refine and validate the conjectures. Often a number of members of the class are involved in generating and refining a given conjecture. Artifacts adopted by the class become a basis for collective reflection and articulation of ideas. Finally, we see classes adopt a stance that knowledge generation is a function of the community and that they do not have to depend on the teacher as the provider and arbitrator of what counts as knowledge.

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