RESEARCH REPORT

NATIONAL CENTER FOR IMPROVING STUDENT LEARNING
AND ACHIEVEMENT IN MATHEMATICS AND SCIENCE

COGNITIVELY GUIDED INSTRUCTION:

A Research-Based Teacher Professional Development Program for Elementary School Mathematics

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COGNITIVELY GUIDED INSTRUCTION:

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Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Franke, Levi, & Empson, 1999) is a professional development program based on an integrated program of research focused on (a) the development of students' mathematical thinking; (b) instruction that influences that development; (c) teachers knowledge and beliefs that influence their instructional practices; and (d) the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of students' mathematical thinking. Our research has been cyclic. We started with explicit knowledge about the development of children's mathematical thinking (Carpenter 1985), which we used as a context to study teachers' knowledge of students' mathematical thinking (Carpenter et al. 1988) and the way teachers might use knowledge of students' thinking in making instructional decisions (Carpenter et al. 1989). We found that although teachers had a great deal of intuitive knowledge about children's mathematical thinking, it was fragmented and, as a consequence, generally did not play an important role in most teachers' decision-making (Carpenter et al. 1988). If teachers were to be expected to plan instruction based on their knowledge of students' thinking, they needed some coherent basis for making instructional decisions. To address this problem, we designed CGI to help teachers construct conceptual maps of the development of children's mathematical thinking in specific content domains (Carpenter, Fennema, and Franke 1996).

In a series of studies (Carpenter et al. 1989; Fennema et al. 1993; Fennema et al. 1996), we found that learning to understand the development of children's mathematical thinking could lead to fundamental changes in teachers' beliefs and practices and that these changes were reflected in students' learning. The studies provided sites for examining the development of children's mathematical thinking, in situations where their intuitive strategies for solving problems were a focus for teacher reflection and discussion. Other studies (Carpenter et al. 1993, 1996, 1998) provided new perspectives on the development of children's mathematical thinking and on the instructional contexts that support that development, which in turn have led to revisions in our approach to teacher development.

In the sections that follow, we describe the CGI Professional Development Program and discuss the research base for CGI with respect to (a) children's thinking; (b) teachers' knowledge and beliefs about children's thinking and the relation of teachers' knowledge and beliefs to their students' achievement; (c) the effect of the CGI Professional Development Program on teachers' knowledge, beliefs, and practice; and (d) the achievement of students in CGI classes. Note that this division does not represent a sequence in which the research was conducted. In fact most of our studies have crossed several categories.

The CGI Professional Development Program

The Cognitively Guided Instruction (CGI) Professional Development Program (Carpenter, Fennema, Franke, Levi, & Empson, 1999) engages teachers in learning about the development of children's mathematical thinking within particular content domains. The theme that tied together our analysis of students' mathematical thinking is that children intuitively solve word problems by modeling the action and relations described in them. By developing this theme, we portray how basic concepts of addition, subtraction, multiplication, and division develop in children and how they can construct concepts of place value and multidigit computational procedures based on their intuitive mathematical knowledge (for elaboration, see Carpenter et al. 1996).

Our engagement with teachers is driven by two principles: (1) we focus interactions with teachers on the fundamental ideas underlying the development of children's thinking about mathematics, and (2) we build on the teachers' existing knowledge. We attempt to provide an environment for teacher learning that offers opportunities for teachers to build on their existing ideas to create continually evolving organizing frameworks of children's mathematical thinking.

Whenever we interact with teachers, be it in a group working session or in a one-on-one interaction, we focus on children's mathematical thinking. We have particular knowledge about the development of children's thinking that we would like teachers to come to understand. In coming to understand this thinking, the teachers create their own ways of organizing and framing the knowledge. They also think hard about the relationship between this knowledge and their teaching. We try not to direct the ways in which the teachers choose to implement their teaching practice. There does not exist one way of implementing CGI. Our intent is not to get teachers to adopt a set of teaching behaviors or moves. Rather, we provide a framework so teachers can think about their students' understandings of mathematics and then make instructional decisions based on the underlying principles. We strive to create inquiry about teaching so teachers are thinking about why they would do certain things and how that relates to the children's learning of mathematics.

Research on Children's Thinking

The model of children's thinking that is the basis for CGI is built on an extensive research base. The research support for our analysis of the development of addition/subtraction concepts was synthesized in Carpenter 1985; Fuson 1992; Gutstein and Romberg 1996; and Verschaffel and De Corte 1993. The research support for our analysis of multiplication/division and the general notion of modeling was reported in Carpenter et al. 1993 and Greer 1992. The analysis of the development of multidigit concepts was supported by research reported in Carpenter et al. 1998, in press; and Fuson et al. 1997.

The results of a study that we conducted with kindergarten children (Carpenter et al. 1993) are summarized in Table 1. In this study we found that, by the end of kindergarten, children in CGI classes could solve a variety of problems by modeling the action or relations described in the problems. Many teachers and curriculum developers considered the problems too difficult for young children, and the study results provided compelling support that children as young as kindergarten can invent strategies to solve a variety of problems if they are given the opportunity to do so. In almost every case, the children used the Direct Modeling strategies predicted by our model of the development of children's mathematical thinking.

TABLE1: KINDERGARTEN CHILDREN'S SUCCESS IN SOLVING VARIOUS WORD PROBLEMS USING EXPECTED STRATEGIES

Problem	*% Who Correctly Solved Problem (N = 70)*
Carla has 7 dollars. How many more dollars does she have to earn so that she will have 11 dollars to buy a puppy?	74
James has 12 balloons. Amy has 7 balloons. How many more balloons does James have than Amy?	67
Tad had 15 guppies. He put 3 guppies in each jar. How may jars did Tad put guppies in?	71
19 children are going to the circus. 5 children can ride in each car. How many cars will be needed to get all 19 children to the circus?	64
Maria had 3 packages of cupcakes. There were 4 cupcakes in each package. She ate 5 cupcakes. How many are left?	64
19 children are taking a minibus to the zoo. The bus has 7 seats. How many children will have to sit 3 to a seat, and how many can sit 2 to a seat?	51

Teachers' Knowledge and Beliefs About Children's Thinking

In a study of teachers who had not participated in the CGI Professional Development Program, we found that teachers had a great deal of intuitive knowledge about children's mathematical thinking; however, because that knowledge was fragmented, it generally did not play an important role in most teachers' decision-making (Carpenter et al. 1988). This study indicated that teachers have informal knowledge of children's thinking that can be built on in the CGI Professional Development Program. In particular, teachers can identify differences between problem types, and they have some idea of many of the modeling and counting strategies that children often use. But most teachers' understanding of problems and strategies is not well connected, and most do not appreciate the critical role that Modeling and Counting strategies play in children's thinking or understand that more than a few students are capable of using more sophisticated strategies.

This study also showed that teachers' knowledge of their students' thinking was related to student achievement. Students of teachers who knew more about their students' thinking had higher levels of achievement in problem solving than students of teachers who had less knowledge of their students' thinking. In a related study (Peterson et al. 1989), we found that classes of teachers whose beliefs were more consistent with principles of CGI tended to have higher levels of student achievement than classes of teachers whose beliefs were less consistent with principles of CGI.

The Effect of Participating in CGI Professional Development Programs on Teachers' Knowledge, Beliefs, and Instruction

In the first CGI study, which investigated the effect of the CGI Professional Development Program on teachers, we focused entirely on addition and subtraction with first-grade teachers (Carpenter et al. 1989). The study was an experimental study in which we compared 20 CGI teachers with 20 control teachers. We found that CGI teachers placed greater emphasis on problem solving and less on computational skills, expected more multiple-solution strategies rather than a single method, listened to their children more, and knew more about their children's thinking than did control teachers.

Whereas the initial experimental study compared different groups of teachers, a three-year longitudinal study of 21 teachers (Fennema et al. 1996) explicitly examined the nature and pattern of change among teachers and the relation between beliefs and instruction. Several levels of beliefs and practice in becoming a CGI teacher were identified: *Level 1* teachers believe that children need to be explicitly taught how to do mathematics. Instruction in their classes is usually guided by an adopted text and focuses on the learning of specific skills. Teachers generally demonstrate the steps in a procedure as clearly as they can, and the children practice applying the procedures. Children are expected to solve problems using standard procedures, and there is little or no discussion of alternative solutions. *Level 2* teachers begin to question whether children need explicit instruction in order to solve problems, and the teachers alternately provide opportunities for children to solve problems using their own strategies and show the children specific methods.

Level 3 is a turning point. Level 3 teachers believe that children can solve problems without having a strategy provided for them, and they act accordingly. They do not present procedures for children to imitate. Children spend most of mathematics class solving and reporting their solutions to a variety of problems. Classrooms are characterized by students talking about mathematics, both to other students and to the teacher. Children report a variety of strategies and compare and contrast different strategies. In sum, Level 3 teachers epitomize the characteristics that distinguished CGI teachers from control teachers in the initial experimental study. Their classrooms are strongly influenced by their understanding of children's thinking, they know appropriate problems to pose and questions to ask to elicit children's thinking, and they understand and appreciate the variety of solutions that children construct to solve them.

What distinguishes Level 3 teachers from Levels 4a or 4b teachers is their use of what they learn from listening to students to make instructional decisions. Whereas Level 3 teachers apply their understanding of children's thinking to select appropriate problems and accurately assess their own students' thinking by listening to the strategies they use, *Level 4a and 4b* teachers conceptualize instruction in terms of the thinking of the children in their classes. Furthermore, they have a more fluid perspective of their students' thinking; they not only apply their knowledge to assess their own students' thinking and to plan instruction, but they also regard it as a framework for developing a deeper understanding of children's thinking in general. In the terms of Richardson (1994), teachers regard their knowledge as a basis for engaging in "practical inquiry." For these teachers, our research-based analyses of children's thinking are not conceived as fixed models to learn but as a focus for reflection on children's mathematical thinking, which helps them organize their knowledge and interpret their students' thinking. These teachers continually reflect back on, modify, adapt, and expand their models in light of what they hear from their students (Franke et al., 1998, in press).

By the end of the study, 19 of the 21 teachers in the longitudinal study were at Level 3 or higher (seven were at Levels 4a or 4b). Eighteen of the 21 teachers had changed at least one level in beliefs and practice, and twelve had changed at least two levels.

In a follow-up study conducted four years after the end of the CGI Professional Development Program, all of the teachers continued to implement principles of the program at some level. Five of the teachers had slipped one level, but ten of the teachers showed continued growth. They not only sustained their beliefs and practices; their learning had become generative so that their classes became places for the teachers as well as the students to learn. What distinguished these ten teachers was that they (a) viewed children's mathematical thinking as central to their teaching, (b) possessed detailed knowledge about their students' mathematical thinking, (c) had well-developed frameworks for thinking about children's mathematical thinking, (d) perceived themselves as creating and elaborating their knowledge about children's thinking, and (e) sought out colleagues for support in understanding children's mathematical thinking.

Case studies (Carpenter et al. in press; Fennema et al. 1992; 1993; Franke et al. in press) also supported the findings of the quantitative studies and provided rich descriptions of teacher change and of the ways teachers have implemented principles of CGI in their classrooms. These studies confirmed the finding of the longitudinal study that change is difficult and takes place over an extended period of time. Developing an understanding of children's thinking provides a basis for change, but change occurs as teachers attempt to apply their knowledge to understand their own students. It is a slow dialectic process, with changes in knowledge and instruction building upon one another. But almost all teachers in our studies have changed in fundamental ways.

The case studies not only showed how teachers can change by learning about children's thinking; they also demonstrated how much can be accomplished by both teachers and students when children's thinking becomes a primary focus for instruction. The studies illustrated how teachers provided an environment in which children's thinking is the focus, children communicated about mathematics, children constructed their own procedures for solving problems, and concepts were developed through problem solving. The case studies described exceptional teachers engaged in the kind of teaching that captures the spirit of reform recommendations and documented how much children are capable of learning in such environments.

Student Achievement

In the initial experimental study (Carpenter et al. 1989), we found that CGI classes had significantly higher levels of achievement in problem solving than control classes had. Although there was significantly less emphasis on number skills in CGI classes, there was no difference between the groups in achievement on the test of number skills. In fact, there was some evidence that CGI students actually had better recall of number facts than did students in the control classes. Additionally, a standardized achievement test, which also measured computation skills, was administered in this study, and no differences were found between CGI and control classes on this test.

In a related study using the same measures, Villasenor and Kepner (1993) found that urban students in CGI classes performed significantly higher than a matched sample of students in traditional classes. Further discussion of the effectiveness of CGI with students from typically under-achieving groups can be found in Carey et al. 1995 and Peterson, Fennema, and Carpenter 1991.

The longitudinal study (Fennema et al. 1996) extended the findings of the initial experimental study. By the third year of the study, the concepts and the problem-solving performances of the classes of every teacher were substantially higher than they had been at the beginning of the study. Improved performance in concepts and problem solving appeared to be cumulative, with students having longer participation in CGI classes showing greater gains in the upper grades during the second and third years of the study. Changes in student achievement reflected changes in teacher practice. For each teacher in the study, substantial improvement in the performance in concepts and problem solving of the teacher's students followed directly a change in the level of the teacher's practice.

Thus, our studies have consistently demonstrated that CGI students show significant gains in problem solving. These gains reflect the emphasis on problem solving in CGI classes. On the other hand, in spite of the decreased emphasis on drill and practice, there is no commensurate loss in skills.

Reviews of CGI by Other Researchers

Extended reviews of CGI appear in several syntheses of research in mathematics education and research in professional development. See for example, Borko and Putnam (1996); Decorte, Greer, and Verschaffel (1996); Ginsburg, Klein, and Starkey (1998); and Wilson & Berne (1999).

For More Information

Educators interested in learning more about CGI can refer to *Children's mathematics: Cognitively Guided Instruction* by Carpenter, Fennema, Franke, Levi, and Empson (1999) and the references that follow.

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