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Equity and Mathematics Reform

Mathematics instruction has been more accessible to students who are members of society's dominant racial, cultural, social class, and gender groups than to those who are not. From a perspective of equity and fairness, all student groups need access to mathematics instruction and opportunities to excel in mathematics. The reform of school mathematics provides the educational community with possibilities for addressing the needs of an increasingly diverse student population. As they develop policy, research, and practice, educators will need to combine concerns for both equity and reform. If they fail to do so, students who do not come from dominant groups may, once again, be denied full participation. The options educators have and the choices they can make are in the areas of curriculum, pedagogy, assessment, and school contexts that facilitate student learning.

Curriculum

If students are to become fully prepared for the world they will inherit, according to a broad consensus in the mathematics education community, the goals of school mathematics must change. Students will require a higher level of mathematical, scientific, and technical literacy than they have in the past. To meet these new goals, the school mathematics curriculum will need to incorporate new content, de-emphasize other content, and draw heavily from real-world and scientific contexts that are linked to out-of-school opportunities.

Existing mathematics curricula ignore students' informal and pre-existing mathematical knowledge and may actively interfere with how they reason and learn. New curricula are being developed with conceptual coherence. These units will focus on learning-by-doing and include problem-solving activities, opportunities for students to justify their solutions to their peers, and other approaches that support students' development of skill in reasoning with mathematics.

While educators agree that school mathematics should prepare students for the opportunities they will encounter in later lives, they reality that these later-life opportunities are not equally distributed in the United States has not been integrated into

the reform agenda. Which mathematical experiences will promote students' interests as individuals and as members of their respective social groups? If school mathematics, for instance, intends to promote full and unfettered participation in a democratic and multicultural society, then the curriculum that American Indian students encounter should help to empower them to manage the lands and resources that are in their trust and to pursue or protect their treaty-guaranteed rights. Students who live in urban areas should encounter a curriculum that will help them use the many resources available in cities and understand and deal with the issues and problems they will face.

In a society that stratifies opportunity on the basis of group membership, there are differential sanctions for the mastery of paper-and-pencil algorithms. New curricula de-emphasize such skills. Some minority communities view facility with paper-and-pencil algorithms as a two-edged sword: On the one hand, ease in using paper-and-pencil algorithms does not guarantee access to opportunity; on the other hand, the lack of mastery is often used to legitimize denying opportunities to people who have been stereotypes as lacking in mathematical competence. From this perspective, the new curricula create a differential risk for students that is based on group membership. The larger society and educators as members of that society must address the differential sanctions but, until then, mathematics educators will need to convince parents of students of color that the new curricula are in their children's best interests.

How best to incorporate meaningful contexts that will support mathematics learning into curricula for diverse student groups is a third area of concern. The multicultural education and ethnomathematics literatures recommend using social and historical settings from around the world as contexts for doing mathematics. Unfortunately, some current efforts to develop a multicultural curriculum treat cultural and mathematical content superficially, recreating stereotypes, and subordinating one type of content to another. Researchers and practitioners are still experimenting and learning about the contexts that are appropriate for teaching mathematics to diverse student populations.

The NCTM *Curriculum and Evaluation Standards* (1989) and the NRC *Everybody Counts* (1989) were informed by knowledge about how students reason and learn, the discipline of mathematics, and the needs of society. The discussion of goals for school mathematics needs to be further informed by empirical knowledge about the social realities that students of diverse backgrounds will face, both as individuals and as members of social groups. Information about census and work force projections, and about the social stratification of opportunities in the larger society will also be necessary.

Pedagogy

After two decades, educational researchers have learned a lot about the teaching methods that help students learn basic number facts and computational skills (Romberg & Carpenter, 1986). There is widespread agreement among mathematics educators that the teaching of mathematics should support student reasoning and engagement in worthwhile content (NCTM, 1991). These pedagogical strategies have demonstrated their effectiveness in improving student performance on a range of instruments (Secada,

1992). New pedagogical approaches are now being developed to facilitate students' mathematical thinking and understanding (Davis, 1992; Putname, Lampert, & Peterson, 1991; Schoenfeld, 1992).

Today's students are more diverse ethnically and linguistically than those whom most of their teachers taught in previous decades. They come from ethnic and linguistic backgrounds that are unlike their teachers' backgrounds and experiences (Grant & Secada, 1992). They may not be middle-class, they may not be White, and they may not speak English fluently. There is little research knowledge about how new pedagogies will affect learners who have been underrepresented in mathematics classes when compared to knowledge about pedagogies used with White middle-class students.

Much knowledge about pedagogy and minority students comes from the literature on multicultural education. James Banks (1993), for instance, argues that quality education for minority students should include integration of content across disciplines, opportunities for students to build their own knowledge, proactive efforts in the classroom and school to reduce prejudice, pedagogical practices that promote equity and inclusion, and an empowering school culture. Studies of teachers who are effective with African American students are showing teachers who care for the students and communicate that caring as part of their teaching (Ladson-Billings, 1994). Effective bilingual teachers use cultural referents to establish the norms for classroom behavior and to relate content to students' backgrounds (Tikunoff, 1985). They also use both languages and attend to English language development within the context of class content.

Work on gender differences in mathematics and on how status differences among students affect their instruction have added insight about the nature of teaching that is good for all students. Elizabeth Fennema and her colleagues have shown that often teachers pay less attentions to girls than boys; ask boys questions that require higher order thinking and reasoning; praise boys more than girls for right answers; and plan competitive learning activities that place girls, who are more likely than boys to be socialized to cooperate, in an uncomfortable position (Fennema & Peterson, 1984). Work on status differences shows that the ideas of students who have high status are given more importance in small group settings; a group often assigns the substantive content of a task to its more capable members (Cohen, 1994). Teachers can take steps to increase their students' access to the mathematics they are taught by paying attention to their questioning of students, by using a balance of cooperative and competitive tasks, but monitoring groups to ensure that all members are participating in the work of the group, and by assisting students who are not participating.

As teachers begin to modify their classroom practices so that they support student understanding, they will need several kinds of support. Research on the school-level reform of mathematics is showing the support teachers provide each other reduces some of the stress of uncertainty that comes when teachers experiment with a new curriculum or try to change a teaching practice. While scholars have realized the importance of teachers' beliefs, assumption, knowledge, and thinking, along with teachers' behavior

and pedagogical practices (Clark & Peterson, 1986; Thompson, 1992), they are only beginning to understand the complex relationships between knowledge, beliefs, and practices. They have not explored the complexity that is added to these relationships when teachers are teaching mathematics to ethnically- and linguistically-diverse learners (Secada, 1991).

Assessment

Assessment and evaluation, according to the mathematics education community, need to be changed to support evolving views of curriculum and teaching. Traditionally, tests have been used to sort persons for educational and life opportunities. A 1993 draft of Assessment Standards for School Mathematics (NCTM) proposed standards that would promote equity by giving students optimal opportunities to demonstrate mathematical power, focusing on worthwhile content, including items that are open ended and require complex forms of thought, and providing teachers with information that can be used to make instructional decisions.

There are lingering questions about bias in assessment. With the possibility that new forms of assessment can make worthwhile mathematics accessible to larger numbers of students, the question of bias takes on added urgency. Will new forms of assessment bring with them new forms of bias that are linked to student demographics? A common strategy for identifying biased items looks for items that fail to predict total test scores or for items with error patterns that are differentially distributed among student groups. As a result, items on which low-achieving students could outperform high-achieving students tend to be thrown out early in the test development process because they do not predict overall test performance.

Contexts for mathematics problem solving are likely to be limited to those that already favor high-achieving students, since contexts that favor low-achieving students may be considered biased and dropped from the test. A pilot version of the California Assessment Program, for example, contained an item that asked students to determine what is wrong with someone assuming that he or she will be accepted to college, given that both College A and College B would accept half of a graduating class. Items set in a college context are unlikely to interest noncollege-bound students; hence, the test that includes them could underestimate these students' performance. Those who develop testing programs will need to determine whether to delete such items because of their potential for bias, or whether to create a smorgasbord of items that includes the same mathematics in different contexts. In the later case, contexts and items that appeal to different groups of students would be included.

The linguistic requirements of new assessments may place mathematics content in a subordinate role. Increased language requirements are likely to dampen the performance of limited English proficient students or others who are not familiar with conventional forms of English. The scoring of student work in mathematics will need to distinguish between mathematical and linguistic competence.

As forms of assessment change, states, districts, and schools will have to reexamine how they use the results they obtain in their accountability or evaluation systems. Differentials in student performance have often been explained by socioeconomic variables. States that report student achievement at the school level have allowed the removal from annual reports of the scores of students who participate in special programs or who possess limited proficiency in English. With increases in the diversity of student populations, schools may feel increased pressure to exclude more students from accountability systems or to attribute changes in scores to factors beyond a school's or a district's control. How to strike a balance between holding individual students, their teachers, programs, schools, districts, and even states accountable, or using external, non-school and non-changeable factors to explain differential performance is the dilemma.

Social Organization of School Mathematics

Social forces and economic considerations influence how classrooms and schools are organized. They also determine how schools create programs to remediate student performance deficiencies and how resources and personnel are distributed. Students have different experiences with mathematics that results from the track or ability group to which they are assigned during their school years. Ability groups are common to the lower-elementary grades; the name implies that students are assigned to them on the basis of ability. Careful analysis shows that, at least in elementary school, ability groups in mathematics often are the same groups developed for reading. Research on the formation of ability groups in reading shows that they are based on teachers' judgments of students' educability, a construct that, in addition to performance, includes students' classroom behavior, and social and emotional maturity. Ability groups can be created within classrooms or between classrooms of students. Their creation is likely to incorporate cultural bias and, according to Slavin (1989) provides no academic benefit in mathematics.

In the upper elementary grades, between-class ability groups become a formal tracking system. While track are often defined by student demographic characteristics such as social class, race, or language ability, they can affect the quality of the mathematics students receive. Less experienced and less able teachers may be assigned to the lower tracks in a system. The mathematics program for these tracks focuses repetitively on procedural as opposed to problem-solving skills and on maintaining classroom order. Students in the lower track are unlikely to encounter the content that students in high tracks receive (Oakes, 1990). The benefits of tracking for high-achieving students may, in fact, be due to differences in the pacing of content of the quality of instruction (Gamoran, 1991).

Chapter I programs that provide bilingual and special educational services are making sweeping changes to align them with curricular, assessment, and systemic reforms. Schools will have leeway to fit these changes into their systems. If students are excluded from a school's mathematics program because they receive categorical services at that time, the program will fail to promote equity in mathematics. Categorical programs will

need to balance their efforts between focused attention for students who are most in need and efforts that address all students.

Schools are microcosms of the society that surrounds them, yet they have unique identities, cultures, and customs. Teachers' efforts to change the way they teach may be limited by the culture and organizational structure of the school in which they work (Little, 1990). While individual teachers work very hard, they often work in isolation from others. Some find support for professional growth and development in their departments or groups of teachers within departments, while others must seek support through extra-school associations and professional organizations (Little & McLaughlin, 1993).

Within a school's walls, its atmosphere, its classes, and its daily routines influence teaching and learning. Do the teachers in that school and department work together to provide a coherent program of instruction where each year builds on teach previous year of instruction? One teacher who teaches mathematics in accordance with reform documents can make mathematics come alive for a group of students. Will these students experience a less inspiring teacher who focuses on the repetition of content using drill and practice methods the next school year? If there are sanctions for students who fail to adapt to different content and approaches, the lack of program coherence will not lead to equity. Persons workings with categorical programs are often unwilling to move their students to mainstream settings because mathematics programs lack coherence and thus differ in their accessibility for students.

The roles of parents and the larger community in supporting of impeding equity and reform have received little attention. The New Math was implemented only in a cursory way because, in part, parents turned against it. Family and home environments contribute to the academic success of children, and, by extension, to their study of mathematics. How a school addresses parental concerns about their children's education will influence the shape of reform in that school and whether all students take part in its efforts. Research is needed on the role that these factors play in diverse learners' success in school mathematics and the strategies that would enable racially, ethnically, and linguistically diverse parents to participate in their children's study of mathematics.

Educators need to know the historical, social, and cultural factors affecting the mathematics education of American minorities. The relationship between the culture of mainstream White America and the cultures of American minorities affects the degree to which members of minority groups are willing and able to cross cultural boundaries, including learning school mathematics and expressing that knowledge in the domains of their lives that are controlled by White Americans. Scholars have revealed the coping strategies African American and Latino students adopt to deal with stereotypic school knowledge, in particular, with mathematics (Fordham, 1988). Now educators must come to understand that the ways they promote the study of mathematics may have the unintended outcome of placing school mathematics outside the cultural frame of reference held by students.

Cross-cultural and historical studies show cultural differences in the mathematical knowledge, attitudes, and behaviors of various minority groups. At present, more is known about the ethnomathematics of schooled and unschooled peoples outside of the United States than of the peoples in this country (Nunes, 1992). It has been assumed by researchers and policy makers that the same mathematical knowledge, attitudes, and behaviors are shared by everyone. Those who do not hold these attitudes and behaviors are considered deficient or in need of intervention programs or activities. As a result of these assumptions, minorities' pre-existing cultural knowledge of mathematics, their mathematics attitudes, and their mathematical behaviors within their cultures or communities have been ignored.

Information about the indigenous systems and beliefs, the situations in everyday life or the activities to which minorities apply and practice their mathematical knowledge, and how the beliefs and practices differ from those of school mathematics is needed. As educators become sensitive to how girls and members of minority groups interpret the mathematical knowledge and practices they encounter in school mathematics classes, they can plan and implement interventions, school based, out of school, or in the larger society, that mitigate against these groups' beliefs that mathematics does not belong to them.

A concern for the mathematics education of all students is grounded in the core of American values of the development of social and intellectual capital, and a consideration for fairness or justice. As the reform of school mathematics proceeds, researchers, practitioners, and policy makers will learn new things about the teaching and learning of mathematics and new things about the nature of equity. The complexity of the intersection of equity and reform will require an in-depth and careful review of options and actions by these groups and members of the mathematics education community.

Note

An earlier version of this article was prepared by the Study Planning Group for Diverse Racial, Ethnic, and Linguistic Groups for the Mathematical Sciences Education Board (MSEB) of the National Research Council (NRC). The study group included Walter Secada, chair, John Ogbu, Penelope Peterson, Lee M. Stiff, and Stuart Tonemah. While this version was written by Walter Secada, it has not been reviewed by members of the study group and neither their endorsements nor the endorsements of MSEB or NRM should be inferred.

Gender and Mathematics Education Research

Five educators who have studied gender and mathematics education over the last two decades, presented a symposium at the April 1994 meeting of the American Education Research Association. The symposium, Research on Gender and Mathematics: Perspectives and New Directions, included Elizabeth Fennema, University of Wisconsin-Madison; Suzanne K. Damarin, The Ohio State University, Patricia B. Campbell,

Campbell-Kibler Associates; Joanne Rossi Becker, San Jose State University; and Gilah Leder, La Trobe University, Victoria, Australia. The five prepared brief papers reflecting their perspectives on the new directions that research on gender and mathematics should take. This article provides abridged versions of the five papers that have been edited to smooth transitions between them.

Elizabeth Fennema

Most research on gender and mathematics during the last 25 years has been conducted from a positivist perspective and has provided powerful and rich information. While, in general, gender differences in mathematics appear to be decreasing, differences between males and females are still found in the learning of complex mathematics, in personal beliefs about mathematics, and in the selection of university majors or careers that involve mathematics. These differences vary by achievement level, socioeconomic status, ethnicity, school, and teacher. Since, in general, teachers tend to structure their classrooms to favor male learners, some interventions that help female learners have been identified.

From 1985-1994 there has been tremendous growth and change in research methodologies. Often grouped together as qualitative, the new methodologies have provided new insight into the complex phenomenon of education. Many researchers have begun to utilize these methodologies and it is sometimes suggested that no more studies of a positivist nature should be carried out. In order to continue the documentation of gender differences in participation and achievement in mathematics, however, some positivist research will be needed. National and international assessments must continue to include to sex of students as a variable, and individual schools must determine gender patterns in the election of mathematics courses and careers. We will not deepen our understanding of gender and mathematics, however, until the scholarly efforts conducted within a positivist framework are complemented with efforts that utilize other methodologies.

While scholarship on gender and mathematics could take many directions, two could help in the identification of important emphases for further research and ensure that women's voices will be adequately represented in educational scholarship: cognitive science perspectives that emphasize the irrelevance of female/male differences, and feminist perspectives that emphasize which female/male differences are critical to the learning of mathematics. Cognitive science deals with mental activity and processing. It is based partially on the major assumption that most behavior is guided by mental activity or cognition. The mainstream of current mathematics education research uses the cognitive-science perspective. Studies of teachers' knowledge and beliefs and of learners' thinking within specific mathematical domains are examples of this perspective. The complexity of the mental processing that occurs as teachers make instructional decisions was revealed using this perspective to study teachers, and the universals of problem-solving behavior that exist across cultures, races, and socioeconomic levels were identified using this research perspective to study learners' thinking. Little research on gender and

mathematics has used this research perspective; most of the studies do not include the variable of sex or gender.

Research on teachers and teaching conducted from a cognitive-science perspective could enrich what is now known about gender and mathematics. Since there is evidence that teachers interact more with boys than with girls during mathematics classes, many assume that if the number of teachers interactions with girls and boys were equalized, gender differences in mathematics would disappear. There is evidence that just counting the number of interactions has resulted in an overly simplistic view about teachers and their relation to gender differences. Studies using the cognitive-science perspective could provide insight into whether teachers make a conscious decision to interact differently with girls and boys.

Studies investigating teachers' thinking have collected evidence that conflicts with commonly held beliefs. One such study (Fennema, Peterson, Carpenter, & Lubinski, 1990) found that teachers thought the attributes of girls and boys who succeeded in mathematics were basically similar, yet their knowledge about which boys were successful was more accurate than their knowledge about which girls were successful. These teachers also attributed the boys' success to ability more than to other sources and the girls' successes to effort more than to other sources. A second study (Weisbeck, 1993) found that while teachers reported they thought more about boys than about girls during instruction, they used similar characteristics when describing girls and boys. Although research that uses a cognitive science perspective is still in its infancy, at least as far as gender and mathematics are concerned, it could provide knowledge about the underlying mechanisms that have resulted in gender differences in mathematics, adding to existing knowledge of overt behavior.

The feminist perspective includes the multiple approaches of feminist methodologies, feminist science, feminist epistemologies, and feminist empiricism. These multiple approaches all focus on interpreting the world and its components from a feminine gender point of view, and the interpretations that result differ dramatically from those that accompany research carried out with other perspectives. Feminist scholars argue that scholarship, including that which developed the fields of mathematics and science, has been carried out by men from a masculine gender point-of-view and incorporates values that are shared by men but ignores those shared by women. While mathematics and science appear to be value free and to report universal truths, both, in reality, are based on masculine values and perceptions.

Feminists in mathematics education are struggling to define what a feminist approach to the study of mathematics might be. Some are examining the ways that females and males think and how they learn mathematics. Some are concerned with using women's voices and their histories to identify important questions. Others are examining the language of mathematics to determine whether it is gendered.

Cognitive science and feminist perspectives, while sharing surface similarities, are based on dramatically different assumptions about females and males. The assumptions dictate

the questions that are developed, the design of studies, and the interpretation of findings. The assumptions are far-reaching and influence how the issue of gender and mathematics is viewed. Are males and females fundamentally different, so that all decisions about mathematics and knowledge about gender and mathematics need to grow from these differences? Or are males and females fundamentally the same, with the exception of their biological differences, and are these differences irrelevant with respect to mathematics teaching and learning? The research community on gender and mathematics must continue to examine the questions that are asked and the research methodologies that currently are used as they go about their business of scholarly inquiry.

Suzanne K. Damarin

Although all feminist research and theorizing begins with the goal of improving the lot of women in the world, feminism is not singular in its underlying assumptions, beliefs, methods, and goals. Feminists work within a range of perspectives and frameworks—liberal feminism, socialist feminism of several sorts, radical feminism, Black womanist theories, and postmodern feminism among them. Most recent research on gender and mathematics is carried out under the assumptions and using the methods associated with liberal feminism which assumes, basically, that the larger structures—e.g., capitalism, the scientific establishment, educational systems, and concepts such as the nature of mathematics, literacy as an essential, and the civilization itself, of current society are stable, essential, and appropriate. Liberal feminists "work within the system," attempting to improve the lot of women within a society that is otherwise left unchanged. Other branches of feminism seek to change the larger system, or in the case of postmodern feminism, to change the ways in which we understand how the system operates.

Several feminists are engaged in an effort to bring the insights, findings, and theories of feminism other than liberal feminism to bear on an understanding of gender and mathematics. This work is characterized by a multiplicity of approaches and the circulation of conclusions that are already tentative and suspect. Like all new paradigms, however, it begins with a rupture from the established ways of conceptualizing, conducting, and interpreting research regarding gender and mathematics. At the root of this movement is a disenchantment with the potential of liberal scientific study to yield solutions to the educational problems and dilemmas of women in relation to mathematics.

Like the field of Women's Studies, the study of gender and mathematics through the lenses of multiple feminism is multi-disciplinary; it calls upon approaches and finds studies in fields that range from the "hard sciences" to cultural studies and the arts. Just as Women's Studies approach to the study of the family might bring together research from sociology, social work, human ecology, medicine, population studies, literature, and religion, a feminism-based study in the area of gender and mathematics might bring together research and scholarship from sociology, educational measurement, history of mathematics in the U.S., studies of the representation of mathematics in the popular culture, mathematical biographies of great mathematicians and of young students, the philosophy of mathematics, and other fields. New data may be gathered from person, documents, or cultural artifacts, chosen in light of the existing discourses and interpreted

to yield new "stories" related to the topic of study. The multiplicity of studies, findings, and stories are read in relation to feminist theory and "against the grain" of each other and of the dominant discourses of gender and mathematics. The purpose is to create a "new reading" of the dominant discourse, a reading that exposes hidden assumptions, unwarranted conclusions, contentious implications, paradoxes, questionable practices, and most importantly, interesting questions.

A feminist analysis of the belief that "math is a male domain," for example, leads to the following: Historical studies reveal that mathematics has long been claimed by men as their domain and philosophers have justified this claim; mathematicians' biographies often make strong masculinized claims about the demands of the field. Sociological studies of women in engineering and related fields of mathematically-based work reveal a high level of sexism in the workplace. Studies of science journalism uncover differences in the representations of scientists, presumed male, and of female scientists that suggest a paradox implicit in the very ideal of a female scientist. Similarly, studies of the popular press document that women and girls are frequently presented as incompetent in mathematics and as aliens in that domain. Even reports of equivalent mathematical performances by the sexes are couched in language suggesting the maleness of mathematics. Taken together and read in light of feminist thought, these findings lead to the conclusions that as a socially constructed area of activity, mathematics is indeed a male domain. In contrast, applications of educational and psychological research treat the belief among women that mathematics is a male domain as a personal attribute, indeed a defect, insofar as it interferes with the desired behavior—e.g., pursuing a mathematical skill. Based on this research, educational efforts are launched to convince young women that mathematics intrinsically is not a gendered field.

In effect, curricula designed and implemented to change this belief ask young women to overcome and/or deny the social realities around them. Would it not be more appropriate to acknowledge that reality in instruction and work to change it? The current goal of enticing individual young women to study mathematics would be replaced by claiming for all women the right of entry to and recognition within the domain of mathematics. This change would move women's claim to the right to mathematics education into the tradition of claims by women to the rights of economic independence, to many arenas of employment, to support for research on women's health, and to many other rights. In gaining each of these rights, education of women to recognize the social issues and education of the general public have been critical. The question, researchable through traditional methods, becomes, Would it work?

This is but one example of the ways that an investigative approach grounded in feminism might change our understanding of gender and mathematics. Virtually all concepts, practices, and assumptions of mathematics education are open to examination through these feminist lenses. Just as mathematics teaching is moving away from the "one right answer" mentality, with the acceptance of qualitative research methods and theories of constructedness and situatedness of knowledge, mathematics education research must also abandon the search for single solutions to complex multidimensional problems. The theories and practices of multiple feminisms, grounded in multiple constructions of sex

and gender, offer to the study of gender and mathematics much knowledge and analysis regarding the workings and functions of sex and gender in society. Careful articulation of the feminist "knowledge explosion" with the knowledge created in two decades of outstanding mathematics education research into questions of sex differences and gender equity promises to yield many new insights, areas of study, and directions for educational research and change.

Patricia B. Campbell

Compared to the 1970's, there has been improvement in the number of girls and women studying mathematics. There are indications, however, that girls' interest in mathematics and their participation in mathematics courses are again beginning to decline. At the same time, the gender gap between top students in science is actually increasing. The apparent stalling of progress has implications for research and perspectives about research. My research looks at programs and strategies designed to encourage or involve girls in mathematics. The assumptions that undergird my research include the following:

Societal and behavioral factors combine to make mathematics unwelcoming to and uncomfortable for many girls and an increasing number of boys. These factors can be identified and changed.

There are weaknesses in both qualitative and quantitative research methods; a single perspective cannot stand on its own.

Bias influences all research methods; the use of multiple methods to answer the same research questions is a reasonable way to reduce bias.

In general, the effects of well thought-out programs designed to increase to participation of girls and women in mathematics are unclear. Once control groups are added, interpretations of results change. In one study, students who completed a summer program, for example, increased their mathematics and science course-taking plans, but so did the students in the control group. Their higher scores after the program on measures of engineering and science-related career interests did not differ from the scores of students in the control group. Results obtained on measures of attitudes were mixed. Responses to open-ended questions showed that after attending the summer programs, girls saw the mathematics they encountered as fun and challenging. A mathematics attitude scale was given for several years, however, and no pre/post changes were found.

In a college study, those applying for a special program were quite different from nonapplicants even though they were enrolled in a similar science major. Some of these differences were fixed, for example, applicants became interested in science at an early age; others were not fixed, for example, applicants had stronger study habits. After two years of college and special-program participation, the groups became more similar and there were no differences in their retention in mathematics/science majors or in their mathematics/science grades. It appears that the college experience had a much stronger impact on these students than the special program experience.

The methods or assumptions used in these studies influenced their results in several ways: The change found in open-ended responses and the lack of change found in attitude scales suggest that subtle effects may not have been observed. By collecting relatively structured data, a number of outcomes and effects were not permitted. Any programmatic results that are obtained without the use of a randomly assigned control group, which often is not used in applied research, must be questioned.

Gilah Leder

This paper explores the influence of choice of a research paradigm on the format and scope of the questions posed for investigation in particular studies: it addresses the issue of coeducational versus single-sex schooling as the means for doing it. Regardless of the research paradigm selected, the findings of a single study can be put into a broader context or their generalizability determined in a variety of ways. Evidence can be assessed: historically—e.g., through comparison with data gathered across time; cross-nationally—e.g., through comparison with findings from the same or comparable institutions in different societies; or cross-institutionally—e.g., by examining data from different institutions in the same society.

Early in 1993, staff at a coeducational high school in Victoria asked for help in assessing the effectiveness of an experimental program they had recently introduced: teaching mathematics to Grade 10 students in single-sex settings. At other grade levels, mathematics would continue to be taught in mixed groups. By selecting Grade 10 for this intervention, the school hoped more females would elect to continue with the more rigorous mathematics courses in succeeding years. No plans were made, either formally or informally, to modify instructional strategies or curriculum materials previously used, or to examine the prevailing culture of the school.

At least two assumptions were implied by the intervention strategy described: that curricula and teaching methods traditionally are geared to the needs of males rather than females, and that content and strategies that facilitate the learning of mathematics for females are more readily achieved in a single-sex setting. Whether broader measures were needed to address to overall context in which learning took place was not considered.

Both quantitative and qualitative techniques were used to gather the data. Attempts were made to document group characteristics as well as the nature of the mathematics traditionally taught in the school, and to determine whether there were any changes in these characteristics over the course of the intervention.

There is no doubt that the design and methods for the research were shaped by the socio/psychological framework adopted in earlier work. There was considerable emphasis on students' perceptions of coeducational and single-sex classrooms but very little on what these organizational settings implied about gender roles or cultural stereotypes. Students' perceptions about their teachers' preferences for single-sex or

coeducational settings were sought; the antecedents or implications of these preferences were not explored. While the nature of the mathematics course was discussed, how or why its content had evolved as it had was not considered.

The segregationist perspective reflected in the design of the experiment appeared to have been adopted by many of the students. They indicated that males and females often behave differently in mathematics classes and that society accepts and possibly encourages the differences. It was generally accepted that teachers would probably interact differently with students in coeducational and single-sex settings. No student speculated spontaneously whether these differences were appropriate, whether the male/female roles they implied should be examined or challenged, or whether the rationale given by the school authorities for mounting the "experiment" should be questioned. These concerns might well have been addressed explicitly if the research had been framed in a different—a feminist—perspective.

Joanne Rossi Becker

This paper takes one feminist perspective on the subject of gender and mathematics derived from *Women's Ways of Knowing: The Development of Self, Voice, and Mind* (Belenky, Clinchy, Goldberger, & Tarule, 1986). *Women's Ways of Knowing* describes a series of stages in knowing that differ in fundamental ways from how men come to know. These stages represent a progression from dependence to autonomy. Learners move through five stages: silence, received knowing, subjective knowing, procedural knowing, and constructed knowing. In the silence stage, the learner accepts authority's verdict of what is true. In the received knowing stage, the learner learns by listening and returns the words of authority. As a subjective knower, the learner depends on what looks or feels right and what comes from her experience rather than an external source.

This seems to be a critical stage for women. The procedural knowing stage has two parts: separate and connected knowing. Separate knowers look to prepositional logic to validate arguments, while connected knowers focus on the context and other people's knowledge. The final stage, constructed knowing, represents an effort to integrate what is known intuitively and what others know. Here the learner appreciates the complexity of knowledge.

Given a characterization of separate knowing as embodying logic, deduction, and certainty, and connected knowing as embodying intuition, creativity, and induction, it appears that mathematics has traditionally been taught to conform more to the former. Could that help to account for the relatively small number of women pursuing mathematics related fields? Research can provide evidence to support or refute the hypothesis derived from *Women's Ways of Knowing* and, if supported, show how this theory can provide and understanding of and improves women's participation in mathematics. The danger exists that acknowledging women's different ways of knowing will serve those (biological determinists) who wish to reinforce stereotypes that demean women's strength and limit their roles, a misuse and misinterpretation of research findings.

To test the hypothesis, in-depth interviews were conducted with 31 graduate students, 17 men and 14 women, in the mathematic sciences. They focused on factors influencing women and men to pursue graduate education. Later these were analyzed through the lens of the *Women's Ways of Knowing* model. One interesting finding was how similar the men and women were in the reasons expressed for liking mathematics and when their interests developed. Of course all like mathematics because they were good at it. They were attracted to the analytical problem-solving aspects of the subject and particularly liked starting with certain assumptions and, through logic, solving the problem. The "objective" nature of the subject also appealed to these informants; they liked being able to determine whether a problem was solved or a proof was correct. This raises a question: Are women in mathematics more likely than female non-mathematicians to be separate knowers and thus to be attracted to the subject because, at least at the student stage, they perceive mathematics to be an objective subject in which they can find absolute truth? Do their views of mathematics evolve as they pursue further study and actually do research themselves?

Nearly all informants developed their liking of mathematics early, in elementary or junior high school. Frequently a teacher was mentioned as one who piqued their interest by providing an enriched curriculum that went beyond arithmetic to problem solving or algebra topics. Thus, it does seem possible for teachers and instruction to make a difference in students' ultimate career choices. Could more extensive use of connected teaching affect more students in this positive way? To better explore whether the model represents how women come to know mathematics, we need further research designed specifically to test the model. This research can be informed by work from several other perspectives. In particular, the segregation perspective makes a case for all-female classes. Does the change in instruction that occurs in such a setting provide a more connected learning environment, which might bring more women into the study of mathematics? Does it enhance students' performance in and attitude toward mathematics? Can a change to connected teaching at all levels recapture women who have been turned off to mathematics and science (Tobias, 1990)?

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Review of NCRMSE Research
Equity in Restructured Schools, by Walter Secada

A majority of people would consider schools equitable if there were no differences on highly valued educational outcomes among students grouped by race, ethnicity, gender or social class. Even if differences were found between such groups, a school could still be considered equitable. If a school is reducing the gap between groups on outcomes, for instance, it can be argued that this school is achieving some degree of equity. Similarly, reducing the differences in how groups of students or their parents perceive their educational experiences can be viewed as a positive step toward achieving equity. Equity, as an idea or a concept, means different things to different individuals or groups. Most people base their definitions of equity on a core notion of fairness, or in the case of the law, on the core notion of justice. In general, they view as inequitable that which appears to be unfair. Hence, a school can work toward equity by applying efforts to

remedy an injustice of by increasing the appearance of fairness or the sense of fairness felt by individuals or groups.

Scholars at the National Center for Research in Mathematical Sciences Education (NCRMSE) and the Center on Organization and Restructuring of Schools (CORS) continue to define equity as the absence of differences between groups on some important outcome measure. They are enlarging this conception of equity to consider the other conceptions of equity that exist is schools. The collaborative research on equity currently underway at NCRMSE and CORS is looking at five sets of interrelated questions:

1. What conceptions of equity do school people use to talk about issues of fairness and how are these conceptions articulated? When they talk about equity, which students do they talk about and how do they talk about them? Which people in these schools hold different conceptions?

2. How do schools, as organizations, enact these different conceptions of equity? Do individual teachers or other school personnel act alone or do they act in concert with others? Are particular conceptions of equity more likely to result in collective activity within a school when compared with other conceptions?

3. What organization features in the restructured schools—cultural or structural—support or impede efforts to promote the different conceptions of equity? How do they operate?

4. How do external agencies influence the conceptions of equity found among school people and the actions school people take to promote equity?

5. How are conflicts or actions involving different notions of equity identified and resolved in schools? Do schools have the resources or other organizational features to support the resolution of such conflicts?

Conceptions of Equity

According to initial analyses of data, six conceptions of equity seem to be held by school personnel. These six form "starting point" for data collection and analysis. It is likely that other conceptions will emerge or that some of the original six will be modified during the course of the research. The six conceptions of equity are labeled: equity as a concern for the whole child; equity as a safety net for individual differences; equity as the same treatment for everyone; equity as compensation for social injustice; equity as triage; and equity as the maximum return on a minimal investment. While the six conceptions of equity have roots in common sense notions of fairness, there are strengths and weaknesses in each of them.

Equity as a Concern for the Whole Child

The first conception of equity grows from a larger idea wherein education is viewed as concerned with the whole child. According to this perspective, each student is an individual who has unique and distinct educational, socio-emotional, and physical needs. In the research, most elementary teachers have expressed concerns about each of the children in their classroom. They feel a deep sense of responsibility and hold themselves accountable for the children's welfare. These teachers are able to give detailed, often heart-wrenching, examples of the actions they take to help a child with academic, emotional, or physical needs.

Equity as a Safety Net for Individual Differences

The conception of equity as a safety net recognizes that a single program cannot meet the educational needs of every student. Hence, teachers and other school personnel who hold this perspective create back-up programs, differentiated curricula, and other resources so that when one program does not work for a particular student, other options are available. Their notions about student and program mismatches often are couched in terms of psychological traits and include learning styles or ability.

Equity as The Same Treatment for Everyone

The conception of equity as the same treatment for everyone seeks to ensure that all children are treated in the same way. This view could be used to justify giving all students a common core curriculum, providing them with similar opportunities to succeed, and holding them to the same performance standards, including those for classroom and school behavior. The argument that everyone should be treated the same is based on the belief that there should be one set of standards for high performance in an area and that society at large demands performance or mastery that meets those standards. All students should be treated the same way so that they have an equal chance to meet the standards and an equal opportunity to succeed in the society at large.

Equity as Compensation for Social Injustice

The conception of equity as compensation for social injustice argues that specific groups of students, for instance a specific ethnic group or females, have not received fair treatment in the larger society or that the groups are not receiving a fair share or the school's resources. From this perspective, the school should actively redistribute resources to remedy the larger social or the more specific school-level inequities.

Equity as Triage

According to the equity as triage conception, schools should divide students into three groups: those who are beyond help, those who, because of special skills or access to special resources, do not "really need the school, since they will make it anyway;" and those for whom the school could provide input that could make a critical difference in whether a student will achieve success or failure in the future. Following the triage

model, school resources would be invested in only the last group, those who fall along the middle of the distribution of the criteria being used, for example, college admission.

Equity as A Maximum Return on Minimal Investment

According to the maximum return on minimal investment notion of equity, schools and teachers should invest in the students who are most likely to benefit from their investment. Given the scarce resources available to schools and stresses that schools, their staffs, and their students face every day, attention and resources should be focused on those students who are most likely to succeed. A school's teachers, for instance, would look for the students whom they view as worth educating. The students would be seen as those who "could be saved." From this perspective, additional resources would be provided to the students at the top of a school's distribution on some indicator of achievement.

Dealing With Multiple Notions of Equity

It is possible for a person or for a school to hold what appear to be competing notions about equity. An individual teacher or a group of teachers may believe that, as far as standards for school discipline are concerned, students should be treated in the same way. Yet the same person or group may believe that the school should provide a range of academic and non-academic programs in order to address the educational aspirations of a diverse student body. Decision-making becomes more complex when multiple notions of equity are applied to the same situation. A school may offer different mathematics courses in an effort to address students' interests and abilities in a fair and equitable manner—equity as a safety net. Such an effort, if taken to an extreme, could result in an ever-increasing number of overlapping courses or the fragmentation of programs. If others in that school believe that the fairest way to educate students is to give them all the same core mathematics curriculum—equity as treating everyone the same—taken to the extreme, it could result in needless rigidity. Such a school would need to find ways to balance the interests of these groups. It would, for example, need to create courses around the same core curriculum but provide students with options as to how they encounter the mathematical ideals and how they demonstrate that they understand what they have learned.

External Influences

External agencies sometimes pressure a school staff to work toward equity in a particular way. One of the schools studied by NCRMSE/CORS was a magnet school created to help a district's desegregation efforts. As its staff developed innovative and highly visible programs over the years, the number of affluent students seeking and gaining entry into the school increased. The parents of the students in this school are now pressuring the school to create programs for gifted and talented students. Teachers who are resisting these pressures say that such programs would take attention and resources away from the school's original mission and lead to differences in the quality of instruction for groups of students.

Another school that was studied is also part of a cluster of magnet schools designed to help desegregate a district. This school's focus includes a program for gifted students that enrolls a disproportionate number of white students. The school says it provides high quality instruction to all students, regardless of the program they are enrolled in. While teachers claim that students in regular classrooms receive the same high quality instruction as students in gifted classrooms, they have been unable to do anything about the size of the classes. The average gifted and talented class has a pupil-teacher ration that is in the high teens while other classes have a pupil-teacher ratio that is in the mid-twenties.

The first magnet school successfully resisted parental pressure because the staff shared the belief that creating a differentiated program for gifted students would lead to inequity. The second magnet school tried to insure a uniformly high quality of instruction across classes; teachers did not talk about re-segregation that was occurring or class size-issues that they acknowledge implicitly could split the faculty into factions.

Managing Dilemmas

Mathematics reform seeks mathematical power and high and rigorous standards for all students. The faculty of both of the magnet schools in the study were familiar with the NCTM Standards, yet they encountered dilemmas. Preliminary analyses of research data reveal features about conceptions of equity in schools. Schools are pulled by competing notions of equity and of actions to promote equity. What a school does that promotes or works against equity is situated in that school's context. Schools and their personnel—even when they are acting in ways that can be considered equitable—are often unaware of the competing principles that undergird their efforts because they do not "talk about it." Can schools achieve equity when teachers act individually to promote their respective visions of equity? Or are there benefits when staff and teachers hold shared conceptions of equity? Schools need knowledge and resources to manage the dilemmas they encounter. The NCRMSE/CORS research is gathering information about how schools that are similar think about equity and how they manage the dilemmas they encounter. It will then identify the variety of options available to schools and the ways that schools can use the options to insure outcomes that are equitable.

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