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In This Issue:

Focus is Middle School Mathematics

Middle school students—those in Grades 5-8—learn mathematics that is very different from that learned in earlier grades. Students in early elementary grades work with the whole-number system as they learn to count sets of objects or to measure attributes. As students move to middle school mathematics, they begin to work with other number systems to represent quantities and magnitudes. Quantities such as common fractions, decimal fractions, ratios, percents, directed numbers, and integers form the core of this content.

The National Center for Research in Mathematical Sciences Education's (NCRMSE) Working Group on the Learning/Teaching of Quantities is examining the growth of student competence in the representation of quantities and magnitudes with systems other than what is considered who number. It has developed a research program that will provide a foundation for instructional and curricular change at the middle school level in the areas of rational number sense, operations on fractions and decimals, and proportional reasoning. Each of the five content-oriented Working Groups of NCRMSE deliberately has chosen to build its research around a content domain rather than grade level or some other feature of schooling. The content-oriented Working Groups are concerned with how students construct an understanding of the major concepts and procedures within a domain, how they apply this knowledge to solve problems, and how instruction influences these processes.

Several present members of the Learning/Teaching of Quantities Working Group held a series of meetings and discussions on the quantities content domain during the period 1987-1990. A summary of the collaborative work of that earlier network of scholars is in the soon-to-be published book, *Rational Numbers: An Integration of Research* (Carpenter, Fennema, & Romberg, in press). One member of that network, Professor of Education Judith Sowder, San Diego State University, reconvened that group and other educators with the same interest in 1991 under the auspices of NCRMSE and now chairs this Working Group.

In addition to Chair Sowder, the Learning/Teaching of Quantities Working Group include principal investigators responsible for work directly related to the group's research agenda, affiliated researchers who carry out activities that contribute to the group's research agenda such as the preparation of a major paper, and affiliated members who may contribute to Working Group meetings or share their work with group members. The group's principal investigators are all from San Diego State University: Barbara Armstrong, Alfinio Flores, Randolph Philipp, and Judith Sowder. Affiliated researchers of the working group are: Robbie Case and James Greeno, Stanford University; Guershon Harel, Pudue University, James Kaput, University of Massachusetts-Dartmouth; Thomas Kieren, University of Alberta; Carolyn Krohn, Stanford University; Susan Lamon, Marquette University; Richard Lesh, Educational Testing Service; Nancy Mack, University of Pittsburgh; Robert Orton, University of Minnesota; Marty Simon, Pennsylvania State University; and Nadine Bezuk, Mary Koehler, Douglas McLeod, Larry Sowder, Alba Thompson, and Patrick Thompson, all of San Diego State University. Affiliated Working Group members are four San Diego-area teachers, Becky Breedlove, Heidi Janzen, Steve Klass, and Sally Movido who assist the group in formulating its research agenda and have an active role in its research; Edward Silver of the University of Pittsburgh, who is a liaison with NCRMSE's National Advisory Panel; and Thomas Carpenter of the University of Wisconsin-Madison, who oversees its work.

Research priorities were developed in early 1991 by Working Group members. Their list included five priorities:

> 1) To interpret classroom activity, content analyses of the various aspects of quantities must be completed; such analyses must be carried as part of ongoing classroom-based research.
>
> 2) To carry out integrated research on teaching and learning, issues regarding a collaboration between researchers and teachers need to be addressed; teachers should be treated as full participants in the development and completion of such research rather than as its potential customers.
>
> 3) To articulate the shared point of view of the group about the teaching and learning of quantities, group members may develop commissioned papers.
>
> 4) To clarify their role in the teaching of quantities, research must address materials and activities used by teachers in their classrooms.
>
> 5) To develop a more specific focus within the domain, case studies of teachers could offer valuable direction.

The conceptual framework prepared for the research of this Working Group was guided by the list of priorities and earlier research reports of group members. While the paper, A Conceptual Framework for Research on Teaching and Learning in the Multiplicative Conceptual field, was developed by Chair Sowder in collaboration with group members located at San Diego State University, it was submitted to all Working Group members

for review. Members reports that received special attention in the development of the framework were chapters in *Number Concepts and Operations in the Middle Grades* (Hiebert & Behr, 1988); *Establishing Foundations for Research on Number Sense and Related Topics* (Sowder & Schappelle, 1989); *Rational Numbers: An Integration of Research* (Carpenter, Fennema, & Romberg, in press); and a paper by Greeno (in preparation), *Notes Toward Semantics of Rational Number*.

Research Projects

Principal investigators Sowder, Armstrong, Flores and Philipp have sought answers to a series of research questions using a case-study approach. Their questions focus on the interaction between teacher knowledge and teacher decisions as well as the connection between teacher decisions and student progress in learning of quantitative reasoning, rational numbers and operations, and ratio and proportional reasoning. In their research they will attempt to answer the following:

A. How does teachers' understanding of content influence the manner in which they teach and what their students learn?

B. As they become more familiar with the mathematics involved and come to understand how students learn the mathematical content,

- What changes and shifts can be noted in the way content is treated by teachers?
- What changes can be noted in the way teachers make instructional decisions?
- What types of topics do teachers view as important for testing?
- How do teachers determine whether student learning is significantly enhanced?
- How do teachers' priorities—e.g. time allocation—change?

Teachers involved in preliminary case studies took part in a series of seminars designed to increase their content knowledge. The content of the seminars was developed around materials prepared by group member Patrick Thompson. Seminar discussions focused on teachers' reactions to items on a content analysis instrument and on frameworks for studying quantities and problems involving quantities. Reports on the preliminary case studies were used as a basis for a program presented at a Psychology of Mathematics Education meeting in August 1992.

The preliminary case studies provided researchers with an opportunity to refine interview and content knowledge instruments. The interviews focus on teachers' planning and decision making, the types of resources they use in teaching, their mathematical and pedagogical backgrounds, and their beliefs about mathematics and the learning of mathematics. Information from these preliminary studies is being used to plan for later studies on measuring teacher knowledge, classroom observation techniques and record keeping, and planning for the professional development of future teachers.

In its next phase, the project will work with a group of middle school teachers who have an inadequate mathematics knowledge base and as a result experience difficulties when teaching middle school mathematics. In the two-year project, content seminars designed for this group of teachers will focus on rational numbers and quantitative reasoning during the first year and proportional reasoning and the development of multiplicative reasoning during the second year. Measures of changes in students' learning will be obtained from the students taught by project teachers during the two-year study. Expert teachers who are part of the Working Group will share planning, teacher development, and classroom observation activities with principal investigators. Members of the Working Group will prepare papers that describe the results of their research efforts.

Affiliated researchers contribute to the Working Group's research by developing papers, by reading and reacting to papers from the principal investigators, and by sharing their work when it related to Working Group goals. Patrick Thompson's (in press) work on quantitative reasoning, for example, included papers, a microworld, and videotapes. It was used with teachers who were part of a preliminary study in the preparation of a series of seminars on mathematical content. The work of Alba Thompson (1991) provided the categories used in the analysis of data on the characteristics of exemplary teachers that is reported in the next pages of this issue of the NCRMSE *Research Review*. Documents by researchers Case, Krohn, and Greeno will be used to plan mathematics content work for future Working Group research.

Affiliated members meet with other Working Group members to discuss theoretical issues and research applications and to review research priorities. National meetings of educational researchers provide opportunities for such meetings. One meeting recently was planned to coincide with the annual meeting of the American Educational Research Association in San Francisco during April 1992; members of the Working Group who were attending the AERA meeting met to discuss the progress of Working Group research and reacted to the papers that were prepared from this its data.

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Reflective Practitioners Reform School Mathematics

By Randolph Philipp, Alfinio Flores, and Judith Sowder

For more than a decade, the National Council of Teachers of Mathematics (NCTM) has guided the reform of school mathematics. Early in 1989 it appointed a commission that produced a set of standards for teaching mathematics. The standards, said the commission, rest on two assumptions (NTM, 1991, p. 2):

- ?? Teachers are key figures in changing the ways in which mathematics is taught and learned in schools.
- ?? Such changes require that teachers have long-term support and adequate resources.

The assumption that teachers are linchpins in the fundamental reform of school mathematics is widespread. Little discussion, however, has focused on the characteristics of teachers who use classroom approaches that embody the vision originally set out by the NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989).

Members of the NCRMSE Working Group on the Teaching/Learning of Quantities began research in 1991 to identify the teacher characteristics associated with individuals considered by their peers as leaders or early adopters—whose mathematics teaching already exemplifies the spirit of the NCTM *Standards*. This article reports on preliminary work with middle school mathematics teachers. During the first year, researchers obtained data on teacher characteristics by conducting extensive interviews, an assessment of content knowledge, and observations during a series of three-hour seminars over a three-month period. They sought information on teachers' conceptions of teaching, their pedagogical and content knowledge, and their teaching behaviors.

While the study set out to identify the characteristics of teacher associated with reform-oriented teaching, early findings also support the assumption that teachers require long-term support and adequate resources. Teacher educators and teacher leaders should find

these experiences replete with ideas for designing programs that would support that efforts of teachers ready to change their teaching behavior.

Teachers selected for the study had often been asked to participate in local and state-level projects for mathematics teachers in curriculum and assessment, research, leadership, and professional development areas. They were active in professional development programs and had established reputations as reform-oriented teachers within the local mathematics education community. They described documents such as the Mathematics Framework for California Public School (California Department of Education, 1985) and the NCTM Standards (NCTM, 1989) as having a profound and liberating effect on them and an incalculable influence on their teaching. For these teachers, the publications seemed to validate their teaching practices.

Conceptions of Mathematics Teaching

In designing their study, the researchers sought a framework for their examination of teachers' conceptions of teaching. They used a recently proposed framework consisting of five categories, each with a developmental perspective (Thompson, 1991). The five categories of conceptions include: (a) what mathematics is; (b) what it means to learn mathematics; (c) what it means to teach mathematics; (d) what the roles of the teacher and the students should be; and (e) what constitutes evidence of student knowledge and criteria for judging correctness, accuracy, or acceptability of mathematical results and conclusions. Data on teachers' conceptions of mathematics teaching were analyzed according to the five categories.

What Mathematics Is

While teachers in this study were not asked specifically what mathematics is, researchers used their answers to indirect questions to obtain evidence of their thinking. Among the indirect questions were those on the kind of mathematics teachers believed should be taught in schools. Rules were not the focus of the mathematics taught by these teachers; they were adamant that school mathematics should not focus on the learning of algorithmic skills. One of their "toughest" jobs, according to these teachers, was structuring their classes so their students learned to discover solutions on their own and to developed their own rules. Many students, they said, thought mathematics involved learning what rule to apply.

Problem solving was considered an extremely important aspect of mathematics by all of the teachers. At times they seemed to view problem solving as one of the many topics by all of the teachers. At times they seemed to view problem solving as one of the many topics in the mathematics curriculum, and at other times they considered it an approach they used to teach mathematics. On teacher commented: "I think problem solving kind of drives what's going to happen. I try to keep my teaching involved around some kind of problem situation."

Another teacher spoke of mathematics as a language that was connected to the real world:

To me, to learn mathematics means to get to a point when you understand that there is no such thing as learning anything in isolation. I tell my kids that mathematics is like a foreign language, and that it's just sort of a language that explains the way things work in the world…to learn mathematics to me is to understand the applications, the usefulness of that language in describing the world around us.

What it Means to Learn Mathematics

Teachers believed that students learn mathematics by being immersed in doing mathematics. They said students learn a great deal of mathematics outside of school that is not recognized in classrooms, "I think the kids have a really pretty good understanding sometimes of quantities, but that doesn't get explored; it doesn't get emphasized at all; it gets almost quashed in the race to learn how to add four-digit numbers…"

Teachers encouraged students to explore and make conjectures and they had them discuss and evaluate the solutions put forward by students during discussions. When teachers prepared activities for their classes they used problems suggested by newspapers or happenings in their schools or as they approached traditional topics in unusual ways. One teacher was amazed that the text devoted only three pages to perimeter and area; this, he thought, indicated students were expected to learn these difficult concepts by looking at examples of regular figures or learning formulas. Yet in his experience:

When students are learning mathematics…what I see is the students who are willing to explore and make mistakes without getting all flustered, students who are willing to not depend on formulas or numbers or things like that in order to solve a problem, students that are willing to talk to each other and with each other about what might work and what might not work.

What it Means to Teach Mathematics

The teachers in this study felt that the development of an understanding of conceptual relationships was an essential focus of their teaching. They were dissatisfied when they, as learners, did not understand the deeper conceptual relationships. Their comments are represented by the following:

My primary focus is understanding. I want those kids to know not just how to do it, but why it works. And sometimes they get tired of that and they want the shortcut, but I really want them to know why it works. I think if you understand why, then you're not going to forget. And you're going to have a background to pursue further.

Teachers felt constrained by curriculum guidelines and textbooks. While their conceptions of appropriate content were tied to their district's curriculum guidelines, some interpreted the guidelines liberally. Either they rarely used textbooks or they used textbooks to cover certain topics at certain points in the year. Most selected other materials and ideas to enrich texts. The goal, according to one teacher, was a "concept-

oriented program where I'm putting together units of concepts rather than just chapters in a book and still covering concepts that I'm supposed to be covering, but hopefully in a more meaningful way for the kids.

Testing programs also affected, to some extent, what teachers taught. Only one of the teachers did not let the standardized tests requirements influence her curriculum choices, "I'm doing some other potentially more meaningful things with these children, and maybe we just don't know yet how to measure these things…And maybe the impact of what I do won't even be known for ten years." Another teacher expressed the conflict he felt in the effort to "Feed into what we are trying to do, the direction we're going (in the reform movement)," yet knowing students "are going to be assessed in some kind of standardized, multiple-choice thing at the end of the year."

Dealing with diversity and individual differences were concerns of all teachers. They did not accelerate their brighter students. The schools where some taught had made conscious attempts to avoid tracking strategies and to mainstream all students. One teacher believed that accommodating differences—cultural, learning, and behavioral—was his biggest challenge, "I kind of keep my expectations at a level where kids are going to be successful if they just do what I ask them to do. I challenge them, but I don't make them frustrated." Another focused on providing students with multiple entry levels:

And, so what I try to really concentrate a lot on is: Am I presenting this information in ways that allow multiple entry levels and allow accessibility by kids of all abilities? And am I trying to teach in a variety of ways? And am I aware? How do I know? What do I do, and how do I know if the kids are not getting it?…I try to be very, very conscious of the reality that all of these people in the room have an incredible variety of prior knowledge and experience, and they're all coming into this learning that I think I want to have happen at exactly the same time with this diversity.

Teachers working in private schools with students from upper socio-economic levels also accommodated student differences. To do so, they avoided closed and narrow mathematics lessons.

The Roles of Mathematics Teachers and Students

The roles of these teachers were affected by their beliefs. To them, the goal of mathematics teaching ought to be the development of children's conceptual understanding of mathematics. They believe that children develop mathematical understanding when teachers provide them with experiences that allow them to do mathematics and ask them probing questions about their work.

Most of these teachers see themselves as facilitators who lead children as they "figure things out on their own." As one put it, a teacher "Can either be—and this is a very old cliché—a sage on the stage or a guide on the side, and when the teacher is a sage on the stage, that's a teacher who's going to have students who depend on the teacher for answers." Students of the sage, he said, grow to depend on the teacher for answers and

hints when stuck and students of the guide grow to see that mathematics problems exist all around them and that there are other sources of mathematical authority than the teacher. He offered the advice, "What the teacher needs to do—and this is really important—is to back off a little bit."

Students' roles can be inferred from these teachers' views of their roles. One put it more explicitly, "I don't have to then tell them that a centimeter is…, I mean 2.54 centimeters makes an inch, because they'll just sort of pick that up. I guess I trust more kids' abilities to be inductive about things than what I was ever taught to do.

What Constitutes Evidence of Student Knowledge

Student responses to verbal questions were the primary means used by these teachers to obtain evidence of student knowledge. An atmosphere of acceptance, they said, is a necessary component of their questioning strategy. This atmosphere also permits students to be honest when they "don't get it." Said one teacher, "I try to have an atmosphere in my classroom of it being wonderful if you're able to articulate what you don't know. Students now that 'why' is going to be the next thing I ask because I really wan them to understand" said one: "Can you explain why this works? Why did we do this? How does this happen? How would you do this and why?"

Observations of students provided these teachers with crucial information about students. In their words, they watched their students' work habits when given problems, their use of manipulatives to model mathematical processes, their selection and use of calculators, and their approaches to problems.

Journal writing was also used by teachers as a means of assessment. Students were asked to keep mathematics journals. The quality of the explanations in their journals provided teachers with additional information.

Although the teachers in this study seemed to attend to the thinking processes of their students, descriptions of their use of assessment information seldom mentioned individual students. Mathematics educators stress using knowledge of students' understanding to make curricular decisions (Carpenter, Fennema, Peterson, & Carey, 1988). Group members appeared to use the information gained through assessment activities to guide their classes as a whole; as one teacher put it, "There's a lot of constant monitoring of where they're at and adjusting what I have to do—kind of based on the way they react and based on the kinds of work I see them do."

Knowledge of Middle School Mathematics

Middle school teachers in this study had a comprehensive knowledge of the mathematics they were teaching and a good understanding of the relevant concepts and procedures. The study had acquired evidence of the mathematical knowledge of these teachers using a written test with a subsequent discussion of responses, interviews, and discussions of problems during research seminars. The written test covered concepts related to rational

numbers, fractions, percents, rations, and rates. Teachers demonstrated a thorough knowledge of the school mathematics appropriate for the grades they were teaching. They had constructed their knowledge in ways that enhanced making connections and using alternate representations and meanings.

Mathematical knowledge, for these teachers, was interwoven with pedagogical knowledge. They knew of numerous ways to teach concepts but made a decision to teach a concept or topic in a certain way because they thought it the best approach for their students. Those concepts and tools that were accessible to their students were of greater interest to them than concepts that required other more advanced mathematical tools. As an example, when presented with the problem,

A 6^{th}-grade student says that 3/8 + 5/12 is 8/20 and justifies her reasoning as follows: "If I made 3 out of 8 free throws in the morning, and 5 out of 12 free throws in the afternoon, then altogether I made 8 out of 20 free throws." How would you respond to that students?,

The teachers had mathematical knowledge that would enable them to explain the problem using algebraic notation and variables, and the concept of weighted averages. They though such an approach was pedagogically inappropriate for 6^{th}-grade students and sought to develop approaches that would be suitable for students at this level.

Teaching Behaviors

Teaching requires planning and online decision making. These complex cognitive skills are applied in the relatively poorly structured yet dynamic environment of classrooms (Leinhardt & Greeno, 1986). Teachers' descriptions of their planning activities and of a typical day in their classrooms were used to obtain data on teaching behaviors.

Long-term planning, undertaken by all of the teachers, was related to their perceptions of their obligation to follow curriculum guides or texts. A majority began with a top-down approach, considering the "big picture" and breaking it down into units and then into individual lessons. One of the teachers approached planning by identifying the "big ideas" and finding a theme into which she could fit the big mathematical ideas. One teacher mentioned using both the NCTM *Standards* and the *California Framework* as well as her experience as a teacher of 3^{rd} and 4^{th} graders to develop broad units of study for her 5^{th} grade class. These units were then divided into weeks of study. Only one teacher used a text-oriented approach while trying to develop a concept-oriented rather than a chapter-oriented focus.

Planning involved sequencing that related to students' understanding of previously learned material. According to one of the teachers, "What is important is that the teacher sees how whatever specifically they are teaching, how it fits into what came before it and what comes after it, and how the child is progressing."

Short-term or day-to-day planning, according to these teachers, often occurred shortly before class of "right on the spot." One teacher described daily planning as "thinking stuff—stuff that I think at night and I think in the morning on my way to school." She wrote out lesson plans but she described the process as "more of a brainstorming thing for me; I'll just write a whole bunch of activities that I've seen, or remember, or think I could do." While they create daily lesson plans, these teachers do not feel and obligation to follow them. A basic outline may be erased, "based on what the reaction to a lesson is one day. Nothing is set in concrete, more like jello."

Resources are essential to the planning processes used by teachers. They are professionals and have gained broad knowledge of their field from reading, gathering materials, and taking advantage of professional development opportunities. One described the materials she had accumulated:

I have been known for a long time as having an incredible resource library. It's too bad I didn't buy stock in Dale Seymour and Creative Publications. And I read all the time, like when the Arithmetic Teacher of the Mathematics Teacher comes, I read it that day…So if you told me to put together some stuff on volume, I could have a year's worth.

A second teacher described how she used her resources:

At first I just sat down and off the top of my head thought about, "What do I want them to learn when it comes to geometry?—and I just wrote down some ideas. Then I start piling all sorts of resources together. I read through the section on geometry in the *Standards*; I read through a lot of curriculum guides on geometry, and through textbooks, to see what they cover. I look at different resource materials, for example, from Marilyn Burns. And I think of ideas for activities. So I have a broad plan throughout the unit of the concepts I want covered, possible activities, and then I sort of plot them in. I throw all my files and all my resource materials on geometry together in a box, and I keep that and look through it for lessons and ideas and take out what I need.

A Typical Day

Teachers in this study described their early teaching as resembling descriptions found in the literature (Romberg & Carpenter, 1986). In the words of one of them:

The first several years of teaching I really was into "This is the section of the book that we're doing today, and here's the practice problems, and now we'll go over the homework, and then I'll teach you how to do it, then you'll practice, and then you'll have some to try before you go home," and that kind of thing.

The teachers teach very differently now, when compared to the model described above. Activities and journals were used by all members of the group. A description of their previous day's class included journal writing toward the end of the class, and it was considered a critical part of their mathematics lesson. One of them commented:

I have (my students) write for a certain length of time and write everything they can think of. Lots of times, if I'm walking around and see some interesting things in their writing, then we'll discuss those as a way to develop the idea we're working on or reviewing from the previous day.

Activities making up lessons are typified by an extensive use of manipulatives. The teachers choose manipulatives carefully, depending on how they would incorporate them into the learning experience. They were familiar with all commonly used manipulatives and indicate they had good classroom supplies, but they sometimes had to order specific types of manipulatives for their schools.

While whole-class instruction is used by all of the teachers, only half of them used cooperative groups as a regular part of the lessons they described. The other half did not think whole-class instruction was optimal and spoke of using more group work in the future. In the words of one, "I would say that most of my teaching is to the class as a whole, which is kind of dangerous. I have been assessing whether that's the best way to do things."

Typical lessons also include problem-solving discussions that often are linked to activities or reports. Going over homework assignments is not perceived by these teachers as an important component of a typical class. They tended to begin a class with some problem or situation related to a previous lesson in order to assess student understanding and then decide whether to give a previous topic more attention before proceeding with the lesson they had planned for the day.

Asked about the major barriers they face when teaching mathematics the way they believe it should be taught, these teachers listed a lack of time and materials, pressure from parents and administrators to make their mathematics instruction conform to traditional methodologies, and the weak mathematical backgrounds of their students. Said one who took a broader perspective, "school is probably the biggest barrier to teaching mathematics…School is the most bizarre place. There is nothing else like it in the whole wide world." Her comments followed a discussion on the ways in which schools partition time, fragmenting the school day.

Descriptions of long-term and day-to-day planning and of typical classes provided by these teachers are reminiscent of the descriptions of preparation and improvisation provided by Yinger (1990, p. 88), "Preparation expects diversity, surprise, the random, and the wild. To prepare is to get ready, to become equipped, and to become receptive." By collecting many ideas and activities they can draw upon, these teachers seem to feel that they are ready to face the diversity and surprises of the day-to-day lessons that are then, to some degree, improvised. Improvisation, according to Yinger, is a highly responsive act that calls for skill and sensitivity to the moment and place. It is structured by actions and dependent on knowledge, beliefs, and goals. To this group of teachers, being well-prepared seems to mean being ready for whatever the day holds, being able to be both proactive and reactive, to be actors without lines who are responsive to their audiences' needs.

The teachers in this study viewed their profession as involving constant growth and change. This view is revealed by their participation in professional conferences and inservice programs, their completion of graduate studies, and their approaches to instructional planning. One teacher said that she and her colleagues had considered putting together a three-year program so that after three years teachers could cycle back through a curriculum. But she and her colleagues decided, "No, we never want to do that. Every year should be different from every other year." Change for these teachers was gradual and ongoing. While they faced barriers in their attempts to reform their teaching, they belonged to strong advocacy groups and continued to seek encouragement and support for their efforts.

The quality of reflectiveness appears to undergrid the changes made by these teachers as they acquired classroom experience. A reflective practitioner of mathematics teaching, according to Houston and Clift (1990), uses knowledge that is pedagogically based, knowledge of students both as groups and as individual learners, an understanding of the milieu of the school and of the community, and an understanding of how all of these interrelate. Reflectiveness was illustrated in the thinking and practices of these teachers. They frequently had ready answers to questions during interviews, giving the impression that they had thought about the issues. They listened carefully and were able to help others articulate their thoughts during seminars. One spoke about lying awake and thinking about the discussions that occurred during seminars. Others indicated they thought about the discussions long after the seminars.

Because teachers can be viewed as key figures in the reform of school mathematics, this study set out to identify the characteristics associated with teachers whose mathematics teaching already exemplified the spirit of the NCTM *Standards*. It also found that the teachers were still struggling to revise their teaching approaches so that they provided problem-solving opportunities for students and incorporated effective ways of responding to their individual differences. Teachers who are reflective practitioners, according to this study, may be more likely to move away from the traditional model of mathematics teaching. In some circles, these teachers would be called heroic in that they sought out their own resources and their sources of support to help them reform their teaching approaches. This finding reinforces the second of the two NCTM assumptions (NCTM, 1991, p. 2), that support and resources will need to be provided if larger numbers of teachers are to change the ways mathematics is taught in the nation's schools.

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Review of NCRMSE Research

The prevailing traditionally-oriented practices in mathematics classrooms contrast dramatically with those envisioned by contemporary reformers of mathematics teaching and learning. A major premise underlying reform efforts is that students are active learners: They do not passively absorb new knowledge, but actively build on their intuitive and informal knowledge and construct new meanings and understandings.

Classrooms that foster active learning involve students in problem-solving activities in a learning community that permits communication among students. Teachers in these classrooms must assume a new and more challenging but also more rewarding role than they previously played.

NCRMSE's Implementation of Reform Working Group has fostered the development of research that describes reform efforts and their impact at classroom, school, district, and state levels. Steven A. Kirsner, an affiliated researcher of the Implementation of Reform Working Group, who is a project manager at the National Center for Research on Teacher Learning, Michigan State University, recently completed one of these studies. The study explored the difficulties faced by teachers who seek to change their roles in order to develop classroom environments that foster active learning.

This review describes and includes excerpts from a paper, Creating A Flexible and Responsive Learning Environment for General Mathematics Students, which reports a portion of the research by Kirsner. An account of one teacher's attempt to create a learning environment such as that portrayed in the NCTM Curriculum and Evaluation Standards for School Mathematics (1989), the paper was prepared by Steven A. Kirsner and Sandra Bethell for the April 1992 meeting of the American Educational Research Association held in San Francisco.

Sandy, a high school mathematics and foreign language teacher, has taught four years and completed three years of graduate education in educational policy and mathematics education areas. Before she resumed her graduate study, she was a fairly traditional teacher. She changed her beliefs about teaching, learning, curriculum, and equity significantly in the course of her work.

Believing that all students are capable and deserve to learn powerful mathematics, she volunteered to teach a general mathematics class of 16 students, 10 of whom had been labeled "special education" students. Her students who had encountered varying degrees of failure and frustration during their previous school mathematics experiences, "had learned that they were not supposed to succeed in mathematics classes. If they were expected to be successful, they certainly would not have been assigned to general mathematics, where they typically would be expected to rehearse the same low-level computations for the third, fourth, or fifth consecutive year."

In an effort to establish an environment where her students could be successful, Sandy attempted the creation of a learning community in which students were encouraged to engage actively in mathematical problem solving and to reason mathematically, orally, and by writing. She employed an eclectic blend of teaching activities, emphasizing manipulatives, students' discussion and communication about mathematical ideas, and groupwork and cooperative learning, as well as individual student seatwork and teacher demonstrations. Rather than using a textbook, she relied on materials she had collected from courses and workshops and wrote her own activities, worksheets, and tests for the 10-week unit on probability and statistics. She justified her unit on the grounds that it would improve computational skills as required by the district curriculum.

Sandy perceived that other teachers in her school saw her as an outsider due to her nontraditional views of teaching mathematics. Other teachers relied on textbooks. They rarely discussed or solicited advice about their own teaching and were unlikely to spend an entire class period exploring one or two problems. Yet Sandy, a Professional Development Teacher, was located at one of the Professional Development School sites that had agreed to work with teacher educators and researchers from Michigan State University to implement a program that stresses collaborative working environments for schools.

Like most of her coworkers, Sandy did not have much time to consult with mathematics educators, given the time constraints on her regular teaching load. While she occasionally consulted with Researcher Kirsner and a special education teacher who was

then a doctoral student in teacher education, she thought that other forms of support were lacking and is confident that increased collaboration and support among her coworkers could greatly enhance her and her students' learning.

Researcher Kirsner observed Sandy's general mathematics class on a regular basis over a four-month period; he also completed interviews with four of her students. A first problem for her 16 students dealt with the probability of having an automobile accident when a driver is either (1) sober or (2) intoxicated, a problem that has significance for most high school students. Students were given data on the number of sober people who have accidents compared to the number of intoxicated drivers who have accidents. Students explored the problem but left the classroom confused about the solution. Sandy suggested that when the goal is learning with understanding, students' may be allowed to puzzle over problems for a longer period of time.

At the end of the activity, according to the study, some students demonstrated facility with fractions in the context of solving a meaningful probability problem "that would not have been evident had Sandy confined herself to teaching fractions traditionally." This finding has been supported by Romberg (1992) who suggests, "thus, present strategies for teaching mathematics by first teaching skills and then exposing students to stylized application problems need to be reversed; concepts and skills should emerge from experience with problematic situations" (p. 37).

Interviews with students suggest that they too sensed the effects of the community of learners established by Sandy. One student, Penny, reflects on group work in an excerpt from an interview. Some of her words have been emphasized by the researchers (**S**=student; **I**=interviewer):

> **I:** What's that been like to work in groups instead of what you're used to?
> **S:** Well, first of all, when I worked in, when I didn't work in groups, it was harder to get to know people. It was hard to do math because the people who know how to do it, we could learn from each other's ideas. But we didn't do it over there. We just worked separated and we actually didn't learn practically anything but what we learned from the teacher. Here we learn from everybody. We learn how they do it, how they understand it and we share our ideas with each other.

Students in Sandy's class had seldom been engaged in academic activities during classes. Observations of Gene, a sophomore student, showed movement from being a disengaged student to being one who became creatively engaged. In the researchers' words:

Gene astounded us by becoming enthusiastically engaged about the mathematical content of a geometry unit. He observed a pattern about polygons, formulated a hypothesis, and tried assiduously to make sense of his observations during the last three weeks of class. What came to be discussed as "Gene's theorem" became the subject matter of at least four class discussions and demonstrations. Although we cannot explain this change in Gene's attitude and behavior with any degree of certainty, we do assert that the classroom environment was (a) supportive of this type of mathematical inquiry and reasoning and

(b) flexible enough to give Gene multiple opportunities to pursue this theorem and refine his thinking with the entire class.

An interview with Gene recounts his views of the effect of a classroom environment that promoted inquiry and learning and encouraged hi inclination to "look for a pattern":

> **I:** All right. How do you explain that all of a sudden you were more interested and engaged? Is there an explanation?
> **S:** It's just—I didn't think that…I didn't think like it before because it just wasn't interesting. And then I found something that puzzled me and I wanted to keep at it.
> **I:** And what puzzled you was the…
> **S:** Every time you add an angle or side you add a 180 degrees. You go from a triangle to a quadrilateral, you add 180 degrees.
> **I:** Is that something that you just thought of yourself or was it—
> **S:** Well, it was on one of our—see were doing polygons and all that. And we had those funny sides for each one of them. And then we had to add up the angles. And she went around the class and they went from 160 to 190, you know around that range. And they went up like about 180 degrees each time. And I just wanted to see if there was a pattern…
> **I:** So for a triangle it is 180, then a quadrilateral was 360, then a pentagon is 180 more than that. 540.
> **S:** 540. Then 720.

While Gene had been reluctant to participate in previous classes, in an environment that promoted problem solving in which he felt safe enough to participate, he showed enthusiasm for a mathematical task.

This study demonstrates that, while there are significant obstacles, a high school teacher can develop a learning environment that promotes learning with understanding. While changing classroom practices is challenging, in this case the rewards of increased student understanding, increased learning oriented discourse among students, and increased student engagement in class activities occurred in a brief four-month period. While the teacher in this study thought additional support would have been helpful, her activities developed an environment that increased learning for a group of students, nearly two-third of whom had been designated special education students.

References

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