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Learning with Understanding

When students acquire knowledge that is connected—rich in linkages among ideas—they understand what they learn. When they understand what they learn, students can retrieve knowledge readily and use it to solve problems in a variety of situations. Over the last two decades, researchers have provided valuable information about understanding and how it develops.

The NCRMSE Working Group on the Learning/Teaching of Whole Numbers is one of seven Working Groups that make up the National Center for Research in Mathematical Sciences Education. Mathematics should be learned with understanding, the group believes, if it is to be useful to students. Starting from the premise that they development of understanding is a basic goal for instruction in mathematics, they reason, instruction should be designed so that students are able to build connections. While it may seem evident that mathematical ideas should not be taught as isolated bits of information, the kind of instruction that is most effective in fostering connections, and thus understanding, is less clear.

The Working Group seeks to learn how the knowledge about understanding which for the most part has been identified outside of classrooms, can be translated into effective instruction. In the next several years, the group will describe and analyze programs of instruction designed to develop understanding in children. Its focus will be on place value and multidigit concepts and procedures. Various instructional programs will be compared and contrasted to gain insight into how instructional components relate to various kinds of learning. The results of the group's work can be used to design new programs of instruction that enable students to learn with a greater degree of understanding. The group, chaired by Elizabeth Fennema and Thomas P. Carpenter, Professors of Curriculum and Instruction in the School of Education at the University of Wisconsin-Madison, also includes principal investigators and affiliated researchers: Karen Fuson, Northwestern University; James Hiebert and Diana Wearne, University of Delaware; Piet Human, Alwyn Olivier, and Hanlie Murray, Stellenbosch University in South Africa. These people are involved in research programs investigating place value and multidigit ideas in instruction.

Work started with the identification of a theoretical rationale which describes parameters that need to be considered as programs are compared and contrasted. The parameters include the goals and assumptions underlying the programs, scope and the sequence of the mathematics to be taught, the role of problem solving, establishing meaning for symbols, the development of skills, coherence within and between lessons, students' articulation of their cognitions, the role of the teacher, teacher beliefs and knowledge, assessment and instruction for individual learners, and classroom climate and discourse (Carpenter, Hiebert, Fennema, Fuson, Olivier, & Wearne, 1991). Each set of investigators will provide a description of their instructional program in terms of the identified parameters. These descriptions will be synthesized so that commonalities and differences of the various instructional programs become apparent.

Measurement of student learning in the four programs will also be done, although this is not a simple matter. The goal is to describe the kinds of understanding that each program develops in students and to be able to relate instructional practice to outcomes. Assessment procedures will be developed to reflect differences in programs, measure the development of fundamental place value concepts and understanding of multidigit algorithms, and assess ability to apply knowledge in unfamiliar contexts. The goal is not to find the "best program" but to compare and contrast the various programs so that in-depth knowledge of how instruction is related to learning with understanding is gained.

Some of the most robust knowledge about the development of understanding is in the domain of whole number arithmetic and the research programs of the group members are investigating the translation of this knowledge into classroom practice. It is difficult to describe adequately the instruction in each program because it emerges and changes as the researchers gain information.

However, the instructional approaches share a number of common components. They utilize research-based knowledge about how understanding is developed and share a common emphasis on the development of learners' conceptual understanding. In each program there is an emphasis on problem solving, on children inventing procedures for solving problems, and on children discussing the strategies that they have used to solve problems. The programs differ in how much explicit instructional support they provide to encourage the development of more advanced understanding in students. Support is provided in some programs by the explicit use of structured physical representations of place value and by discussions of the potential linkages between symbols and objects. In other programs, teachers decided whether or not to use materials depending upon their perception of the child's understanding.

To illustrate the work of four different programs, consider how meaning is established for place value and for symbolic procedures or algorithms for addition and subtraction. One research program, directed by Karen Fuson of Northwestern University (Fuson, 1986, 1990) investigates linking children's developing conceptions of place value with operations on concrete materials. Teachers are given explicit instructional materials to use. In these materials, students are led to connect or link symbols to external representations, base-10 blocks, that embody the quantitative values that are only implicit in the written base-10 system. Thus, the blocks can help children understand and use the quantities in the written place value system. The blocks can be manipulated in a way that corresponds directly to the steps in standard addition and subtraction algorithms, or they can be used by children to invent their own procedures to solve problems (Fuson, Fraivillig, & Burghardt, in press).

When using base-10 blocks in a linking approach, meaning is given to computational algorithms by connecting each step in the algorithm to corresponding actions on the blocks. Consider the subtraction algorithm which requires regrouping. Blocks first are used to represent the large number. To subtract, 10 for 1 regrouping or exchanging is required so that there are enough unites within each place for the appropriate number to be removed. When the exchange is made with blocks, the corresponding regrouping marks are noted with the written symbols. Thus, quantitative exchanges with the blocks are linked directly to regrouping in the symbolic algorithm.

Fuson currently is investigating culturally appropriate ways for instruction to support the conceptual development of urban Hispanic children as they develop abilities to solve multidigit addition and subtraction problems. This support involves the linking procedure described above as well as the use of "tens words" for all numbers larger than one digit. Tens words are English versions of Chinese number words; they make explicit the place value concepts embodied in spoken number words by specifically naming the tens, e.g., 53 is said as "five tens and three" and 14 is said as "one ten and four." The linking and use of tens words are embedded in a classroom environment of problem solving in which children individually or in groups solve problems and then discuss alternative solution procedures (Fuson & Fraivillig, in press).

A second instructional research program dealing with place value is that of James Hiebert and Diana Wearne at the University of Delaware (Hiebert and Wearne, 1992). They structure their teaching of place value as Fuson does, by helping students connect symbolic notation with base-10 blocks. Teachers help students establish meaning for symbols by building connections with the referents and then encouraging students to develop strategies for adding and subtracting using the referents and the written symbols. Strategies are worked out in the context of problem situations that involve both addition and subtraction.

In the Hiebert and Wearne project, procedures for manipulating symbols are built on the meanings students have established for the symbols. More complicated algorithms are worked out by elaborating previously constructed symbolic procedures. Teachers ask students to share and analyze alternative procedures, including the standard algorithm. Students defend and justify procedures based on the meanings of the symbols and on analogies to referents.

Two instructional research programs which are somewhat different from the previous two are the Cognitively Guided Instruction (CGI) project at the University of Wisconsin-Madison directed by Elizabeth Fennema and Thomas Carpenter (Carpenter and Fennema, in press), and the Problem Centered Primary Mathematics Program (PCM) at Stellenbosch University in South Africa directed by Piet Human, Alwyn Olivier, and Hanlie Murray (Olivier, Murray, and Human, 1990). In both of these programs, students are asked to use their existing knowledge about place value and solving one-digit problems to invent procedures for solving problems involving multidigit numbers.

The CGI and PCM programs differ somewhat on instruction with standard algorithms once children are proficient with invented strategies. In the PCM program, young children use only their own invented algorithms. A fundamental principle is that children should never believe that they are compelled to use any specific procedure. Standard algorithms are delayed until

Grade 4 or 5. Because the CGI program is based on teacher decision making, once students are reasonably proficient in constructing their own procedures for solving multidigit problems, standard algorithms are sometimes introduced as more efficient procedures that provide a way of recording and keeping track or steps in a systematic way. Thus, the standard algorithms are connected to place value concepts through the invented procedures.

The Whole Number Working Group will not identify an optimal program, one that is most effective in teaching for understanding. The group believes that all instructional programs that promote understanding, although different in some ways, may include significant components which are similar across programs. Thus, it hopes to identify and describe these instructional components which are essential for the development of all students' understanding of mathematics.

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Cognitively Guided Instruction

CGI is a philosophy versus a recipe…You as a teacher have to take the knowledge that CGI is about problem types, about solution strategies, about how children develop cognitively and you have to apply that to your own teaching style. (Mazie Jenskins, First Grade Teacher)

A CGI classroom is where you build on the math knowledge of your children according to what they know…You don't build objectives that say they should be doing this, this, this, and this. You sort of take what they know and build from there. (Susan Gehn, First Grade Teacher)

Cognitively Guided Instruction (CGI) is captured in the above statements of experienced CGI teachers; it is about teachers making instructional decisions based on their knowledge of individual children's thinking. Currently in its seventh year of funding from the National Science Foundation, CGI was started by Elizabeth Fennema, Thomas Carpenter, Penelope Peterson, and Megan Franke as a research program to investigate that impact of research-based knowledge about children's thinking on teachers and their students. The project currently includes the investigation of childrens' and teachers' thinking in Kindergarten through 3rd grade, the study of CGI in urban schools, and the study of the impact in CGI in preservice teacher education. Teacher education materials are being developed for use with both inservice and preservice teachers.

What CGI Is

CGI is not a traditional primary school mathematics program. It does not prescribe the scope and sequence of the mathematics to be taught. Nor does it provide instructional materials or activities for children, or suggest that there is an optimal way to organize a class for instruction. Instead, knowledge about mathematics, in terms of how children think about the mathematics, is shared with teachers. The learning environments of CGI workshops are structured so that teachers learn how the knowledge about children's mathematical thinking can help them learn about their own children. With CGI support, teachers decide how to use that knowledge to make instructional decisions. As teachers implement and reformulate the plans they make, unique CGI classrooms emerge. Each teacher creates a teaching and learning environment that is structured to fit his or her teaching style, knowledge, beliefs, and children.

Even though CGI does not prescribe instruction, CGI classrooms do have similarities. Children in CGI classrooms spend most of their time solving problems, usually problems that are related to a book the teacher has to read to them, a unit they may be studying outside of mathematics class, or something going on in their lives. Various physical materials are available to children to assist them in solving the problems. Each child decides how and when to use the materials, fingers, paper and pencil; or to solve the problem mentally. Children are not shown how to solve the problems. Instead each child solves them in any way that s/he can, sometimes in more than one way, and reports how the problem was solved to peers and teacher. The teacher and peers listen and question until they understand the problem solutions, and then the entire process is repeated. Using in information from each child's reporting of problem solutions, teachers make decisions about what each child knows and how instruction should be structured to enable that child to learn.

Starting at the kindergarten level, CGI teachers ask children to solve a large variety of problems involving additions, subtraction, multiplication, or division. Children learn place value as they invent procedures to solve problems that require regrouping and counting by 10s. Little work is focused explicitly on the mastering of counting, basic facts, or computational algorithms. Instead problems are selected carefully so that children count by 1s, 10s, or 100s depending on the child; discuss relationships between basic number facts; and invent procedures to solve problems involving two-and three-digit numbers.

The climate in a CGI classroom is one in which each person's thinking is important and respected by peers and teacher. Children approach problem solving willingly and recognize that their thinking is critical. Each child is perceived by the teacher to be in charge of his or her own learning as individual knowledge of mathematics is used to solve problems that are realistic to her or him. Mathematics is usually taught at least an hour a day; it is also integrated into the many other learning activities children do.

Author Notes: Work on children's thinking in geometry and measurement began with CGI teachers in January, 1992; the results have not yet been implemented in classrooms.

Knowledge about Children's Thinking

Recent research has provided a reasonably clear picture of how many basic mathematical ideas develop in children. This research has shown that children enter school with well developed informal or intuitive systems of mathematical knowledge that can be used as a basis for the further development of their understanding of mathematical concepts, symbols, and procedures. Even before children are introduced to formal notions of addition, subtraction, multiplication, and division—they can solve a variety of problems involving the actions of joining, separating, comparing, grouping, partitioning, and the like.

To understand children's intuitive problem solving processes, it is necessary to understand the different problem situations that characterize addition, subtraction, multiplication, and division. Therfore we start by helping teachers develop a taxonomy of problem types. Some of the distinctions among subtraction problems and division problems are illustrated by the problems in Table 1.

Table 1

Addition, Subtraction, Multiplication and Division Problems

1) Sybil had 12 stamps. She gave 8 of them to George. How many stamps did Sybil have left?

2) Sybil had 8 stamps. George gave her some more and then she had 12. How many stamps did George give her?

3) Sybil had some stamps. George gave her 8 more and then she had 12 stamps. How many stamps did Sybil have before George gave her any?

4) Sybil had 12 stamps. George had 8 stamps. How many more did Sybil have than George?

5) Sybil had 12 stamps. She put 4 of the stamps on each page of a book. On how many pages will she put stamps?

6) Sybil had 12 stamps. She wants to divide them so that she and 3 friends have the same number of stamps. How many will each person get?

The distinctions among the problems in Table 1 are critical because they reflect the ways that children think about and solve problems. Initially children directly model the action or relations in the problem. They solve the first problem in Table 1 by making a set of 12 counters (to represent the stamps) and removing 8 of them. They solve the second by first making a set of 8 counters and then adding more until there are 12. The fourth problem is often solved by matching a set of 12 counters with a set of 8 counters. Many children cannot solve the third problem because it is difficult to model. They have no place to start because the initial set, i.e., how many stamps Sybil had to start with, is not known. The fifth problem is solved by putting 12 counters into groups of 4 and counting the number of groups. The last problem is solved by first dealing 12 counters into 4 groups, and then by counting the number of objects in each group. With the possible exception of the third problem in Table 1, all these problems can be solved by many children who are as young as Kindergartners of 1st graders, if they are given an opportunity to model the problem situations.

These modeling or concrete strategies provide a foundation for the development of more abstract ways for solving problems and thinking about numbers that involve counting. For example, children come to recognize that it is not necessary to make a set of 8 objects to solve the second problem. They can find the answer just by counting from 8 to 12 and keeping track of the number of counts. Similarly, the first problem can be solved by counting back 4 from 12, and the fifth problem can be solved by counting by 4s (4, 8, 12). The sixth problem, on the other hand, is more difficult to solve by counting. Since the number of objects in each group, i.e., the number of stamps each child is to receive, is not known, children do not have a specific number to count by. Thus, this analysis of problems and children's strategies for solving them provides a principled basis for understanding differences in the difficulty of the problems and why children may have more difficulty in using particular strategies to solve certain problems.

In the process of solving problems, children learn number facts, not as isolated bits of information, but in a way that builds on the relationships between facts. Certain facts like doubles, e.g., 6 + 6, are learned earlier than other facts, and children use this knowledge to solve problems and to learn other facts. For example, consider how one child figured out that 8 + 9 is 17, "Well, 8 and 8 is 16 and 8 and 9 will be just one more. So it's 17."

Knowledge of place value and computational algorithms also can be developed through problem solving. The counting and modeling solutions that children use with smaller numbers are extended naturally to problems with larger numbers. Rather than using individual counters and counting by 1s, children use physical representations for 10s and 100s to model two-and three-digit numbers. They learn to use symbols by inventing procedures for solving two-and three-digit problems without counters as illustrated in Table 2.

In summary, children start school with a conception of basic mathematics that is much richer and more integrated than that presented in most traditional mathematics programs. For example, in most textbooks subtraction is presented as only a separating or take-away action. However, subtraction can also be represented by the comparing, joining, and part-whole problems illustrated in Table1, and young children can solve such problems. Symbols, then, are learned not as abstractions, but as a way of representing situations that children already understand. Rather than expecting children to learn skills in isolation and then to learn how to apply those skills to solve problems, the learning of computational procedures is facilitated by problem-solving experiences that permit children to invent ways to calculate answers to problems.

What Research has Shown About CGI

Previous studies have investigated whether the CGI knowledge that we shared in the workshops had an impact on teachers and on students; the results have been reported in a variety of publications. The studies have used a variety of methodologies to study teachers including precise observations of teaching, paper and pencil assessments, individual interviews, and in-depth case studies. To assess children's thinking, standardized tests, self-developed paper and pencil tests, and individual interviews have been used. The majority of the studies that have been reported to date have been concerned with the learning and attitudes of 1^{st} grade children and with the thinking and instruction of their teachers. The findings from a number of studies that have been conducted over the last seven years are synthesized here.

Teachers and CGI

In general, teachers can learn the knowledge about the mathematical domains and children's thinking within those domains. The knowledge has proved useful to them. They are able to use it as they plan for and implement instruction and to assess what individual children know. Teachers can use this knowledge to make instructional decisions, both before and during instruction.

The instruction of teachers who have been through a CGI workshop is different than the instruction of teachers who have not been exposed to CGI, and children in CGI classrooms do different things when compared to those in non-CGI classrooms. When compared to non-CGI teachers, CGI teachers assess their children's knowledge more often and use a larger variety of procedures to gain knowledge about children. Much assessment is integrated into ongoing instruction, when the teachers gain knowledge of children by asking questions and listening to their children's responses. Some teachers supplement this informal assessment with individual interviews. Mathematics is integrated throughout the day and problems are situated in a variety of contexts which have meaning to children. Teachers find mathematics becomes more fun to teach when CGI principles are used.

Successful Implementers of CGI

While all teachers who have participated in a CGI workshop appear to change their instruction, some teachers are better able to implement CGI than others. A number of studies have identified teachers who appear to implement CGI better than others using some kind of inter-rater judgment or by measuring the learning of children in the classrooms. The two sets of teachers' characteristics have been compared and contrasted and the relationships between these characteristics and their children's learning examined.

On characteristic critical to any implementation of CGI is the knowledge that teachers have: knowledge of content analyses and children's thinking in general, as well as knowledge of the thinking of specific children in their classrooms. Before any exposure to CGI, many teachers have an intuitive knowledge of content analyses and how children solve problems. However, that knowledge does not appear to be particularly well integrated and organized. It is not particularly useful to them as they make instructional decisions. After participating in CGI workshops and using the knowledge as they teach, the knowledge becomes integrated into a more coherent network and used as a basis for making instructional decisions. The knowledge of the better implementers of CGI is more highly integrated than the knowledge of those who implement it less well. The degree of knowledge that teachers have about CGI and their children's learning is correlated with what their children learn in mathematics.

Teachers' beliefs about mathematics instruction, i.e., their role and students' role in learning mathematics, is another important characteristic. Those teachers who hold beliefs more closely aligned with the philosophy of CGI are better able to implement CGI. The degree to which these beliefs are held is positively correlated with the children's learning.

More successful CGI teachers believe more strongly than less successful CGI teachers that: 1) children's learning should be considered as they make instructional decisions; 2) children have informal knowledge that enable them to solve problems without instruction; 3) the teacher's role is to build a learning environment where children can construct their own knowledge rather than where the teacher is a transmitter of knowledge; and 4) the learning of procedural skills does not have to come before children can solve problems.

Becoming a CGI teacher is not done overnight, nor is it accomplished by the end of a workshop. It takes time and interaction with children to learn CGI knowledge, and to incorporate it into a classroom. The more the knowledge is used to gain an understanding of individual children's thinking and ability, the more important it becomes to teachers. They increasingly ask questions that elicit children's thinking, listen to what children report, and build their instruction on what is heard. Teachers increasingly come to believe in the importance of children's thinking as they see what children are able to do and what they are able to learn when given the opportunity to engage in problem solving appropriate to their ability.

Children and CGI

The learning and beliefs of children who spent one year in a classroom taught by teachers who had attended a CGI workshop have been compared with those of teachers who had no CGI education. Children in the CGI teachers' classrooms spent more time solving problems and talking about mathematics with their peers and teacher and less time working on computational procedures than did children in non-CGI teachers' classrooms. They reported more confidence in their ability to do mathematics and a higher level of understanding than did non-CGI students. When compared to non-CGI students, children in CGI classrooms were better problem solvers; in spite of the fact that they spent only about half as much time explicitly practicing number fact skills, they actually recalled number facts at a higher level than did non-CGI students.

CGI children become more flexible in their choice of solution strategies and increased their fluency in reporting their mathematical thinking. Children in CGI classrooms learned much more than has been expected of children in traditional classrooms. They learned to solve a larger variety of addition/subtraction and multiplication/division problems; their understanding of place value increased; and they learned to be flexible in their use of invented strategies to solve multidigit problems.

CGI In 1992-The Longitudinal Study of CGI in the Primary School

The purpose of current CGI research is to study the impact of providing primary teachers with access to a structured, coherent body of knowledge about children's thinking in mathematics on teachers' knowledge and beliefs, their instruction, and their students' learning over a three-year period. Research based knowledge about children's thinking in addition/subtraction, multiplication/division, place value, early ideas of fractions, geometry, and measurement has been identified. Workshops have been developed and taught to most of the Kindergarten through 3rd grade teachers in participating schools. Currently how teachers come to understand their students' thinking, how teachers use children's thinking to develop and provide instruction, the impact of the knowledge of children's thinking on teachers' knowledge and beliefs, and the cumulative effect of being in CGI classrooms for three years on students' mathematics learning are being studied. Additional studies are looking at the scope, sequence, and pedagogical presentation of mathematical ideas by two different, expert CGI teachers per year to obtain rich descriptions of CGI in Grades 1-3.

The Development of CGI Educational Materials

The materials that were written to enable CGI to be implemented are being revised into a coordinated program which can be used with either preservice or inservice teachers. These materials will include chapters detailing CGI philosophy; the content analyses of addition/subtraction, multiplication/division, place value and multidigit algorithms, functions, and geometry; children's thinking; and video tapes that illustrate children's thinking and prototypic classrooms. If feasible, hypermedia will be made available to help teachers interact with CGI ideas. These CGI educational materials currently are being tested using a variety of procedures to organize workshops. Descriptions of the procedures will be made available to assist future workshop developers.

CGI in Urban Settings

Under the direction of Deborah A. Carey, a further study of CGI's impact on 1st grade children and their teachers is underway in six magnet schools with 60 percent or more racial/ethnic minority populations in Prince George's County, Maryland. Of particular interest is the change is teachers' expectations as they learn to assess their children's knowledge. The change in instructional behavior which happens as teachers learn to use children's knowledge is being documented. Both quantitative and qualitative research methodologies are being used. Any modifications of workshop materials which are necessary when working in multicultural settings will be noted and incorporated into the teacher education materials that are being produced.

CGI and Preservice Education

A further investigation is studying the conditions under which CGI can be incorporated into preservice teacher education programs. Of interest is whether and under what circumstances successful inservice teacher education programs can have similar effects on preservice teachers. This study, directed by Donald Chambers, is a collaboration with teacher education faculty at Queens College, City University of New York; San Diego State University; and the University of North Carolina at Greensboro, and the primary-grade teachers in schools used by those institutions for field-based preservice teaching experiences.

The first phase of this project, beginning in February 1991 and lasting about 18 months, is designed to help the teacher education faculty and primary-grade teachers at each of the three sites develop sufficient expertise in CGI knowledge and its application in primary-grade classrooms so that they can serve as experts in the education of preservice teachers. Activities during this period will include workshops at each of the three sites, a two-week summer conference in Madison, Wisconsin, and visits to the classrooms of the site team teachers. Not only are the project members at each site becoming familiar with the application of the principles of CGI, but they are also planning for implementing CGI into their preservice teacher education programs through university course modifications and field assignments. Some piloting of instructional activities is also taking place at this time. Workshops are being conducted at two sites to expand the cadre of CGI mentor teachers available to supervise the field experiences of preservice teachers.

The actual intervention will begin in the fall of 1992. Team members at each site are developing the site's intervention plan which will be finalized during the second summer conference in July 1992. Plans for the evaluation of the impact of CGI in ther preservice teacher education program are also being developed for each site. The evaluation will look at changes in the beliefs and knowledge of preservice teachers and their ability to use CGI principles in their interactions with students. Interventions will be documented and modified over a two-year period in an attempt to achieve the maximum possible impact.

The results of the study will be published in a monograph that will include an overview of CGI, descriptions of the activities and results at each of the three project sites, an evaluation of each site, a cross-site synthesis, and suggestions for the incorporation of CGI into teacher education programs at other institutions. This monograph should become available in the spring of 1995.

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The bibliography contains only a partial list of CGI publications. A complete list of CGI publications can be obtained by writing NCRMSE.

Review of NCRMSE Research

The mathematics education community reached the conclusion several years ago, "that all students need to learn more, and often different, mathematics and that that instruction in mathematics must be significantly revised" (NCTM, 1989). Under the aegis of the National Council of Teachers or Mathematics (NCTM), a cross section of that community developed standards for a reformed school mathematics curricula in 1987-1988. Their standards can be found in *Curriculum and Evaluation Standards for School Mathematics*, a 1989 publication of NCTM.

NCRMSE invited scholars who were developing or working with innovative mathematics programs or materials that possessed features related to the Standards to a 1988 meeting. Participants at that meeting prepared proposals to study the programs or materials when used by teachers in United States classrooms. A proposal of the Freudenthal Institute, University of Utrecht, The Netherlands, was funded by NCRMSE. The proposal involved the preparation of a prototypic unite by Dutch mathematics educators, and a trial of the unit with six teachers and their algebra classes at Whitnall High School, Greenfield, Wisconsin.

This review summarizes a publication on the Whitnall study, *Learning and Testing Mathematics in Context: The Case of Data Visualization* (de Lange, van Reeuwijk, Burrill, & Romberg, in press). It covers the basis for the unit, the actual unit, the method used for the study, and the results of the study. The unit, *Data Visualization*, has been published (de Lange, Verhage, 1991).

Realistic mathematics education is the term used by the Freudenthal Institute, University of Utrecht, to describe the type of textbooks adopted by most Dutch schools. According to the director of the institute, Jan de Lange, more than 50 percent of primary level textbooks could, in 1985, be classified as realistic and, at present, four out of five newly selected books are realistic in content. The unit on data visualization was developed for the study from the realistic view that students need to experience real-world situations or problems. In the view of its Freudenthal Institute developers, concrete situations allow students to apply the mathematics they know, and in effect, to mathematize situations. It is from such experiences that students develop an understanding of mathematical concepts.

The unit on data visualization is designed to assist students in the development of the skills required to use critically the statistics presented by the media. Today's adults encounter visual representations of data in newspapers and on the television on a daily basis. The Standards state that knowledge of statistics is necessary if students are to become intelligent consumers who are capable of making informed and critical decisions. The unit includes activities designed to teach students about describing a data set numerically, representing data graphically, and examining representations of data critically. Using tables and graphs found in current newspapers and magazines, the unit examines presentations and conclusions about population demographics and presidential elections, the relation between education and income, running speed and oxygen consumption, crime, and cholesterol.

The unit exposes students to a series of problems build around a context or theme. Students interact with the teacher and other students as they begin work on the problems. The teacher does not tell the students how to solve the problems in a traditional sense, but leads or guides the students through the instructional activities, monitoring their progress and creating opportunities for them to share their approaches or to discuss the relative merits of their solutions. Assessment activities are designed to motivate students by providing them with feedback on their progress. The activities fit the guidelines provided in the *Standards* (NCTM, 1989, p. 191).

The unit is constructed so that it elicits students involvement. Early activities are designed to foster students' intuitive exploration of the field but later activities require a more reasoned approach. Each section's activities give students new techniques while requiring they use some basic arithmetic skills. The topic is treated in a general way in the first four chapters, but in the last three chapters, students begin to learn more specific content such as box-plots, stem-leaf diagrams, and scatter diagrams. Each of the unit's chapters addresses lower order cognitive goals, but also includes open questions that address higher order cognitive goals. The interplay between intuitive and reasoned, new and old, general and specific, and lower and higher goals give all students the opportunity to experience success with some portion of the content throughout the unit.

Seven Whitnall High School algebra classes were involved in a five-week trial of the unit on data visualization. The seven included one honors, four 9^{th} grade, and two 10^{th} grade classes. An observer from the Freudenthal Institute met with the six teachers of the classess before they began to use the unit and—on a weekly basis—while they were using the unit. During these meetings, the teachers and the observer worked through and discussed the activities. The observer also sat in on three of the classes on a regular basis and the remaining four on an intermittent basis during the trial period. Students completed an attitude questionnaire during their initial session with the unit. They also completed several assessment tasks during the study.

The intent of the study was to identify the kinds of problems schools, teachers, and students would face when implementing the vision of school mathematics implied by the Standards. While the unit was designed to encourage students to explore a mathematical domain under the guidance of a supportive teacher, the observers concluded that the unit was a necessary but not a sufficient element for implementing the vision. Those carrying out the study underestimated the kind and the amount of teacher preparation necessary before the start of the trial. It appeared that, regardless of how conscientious and well educated teachers are, they must know, understand, and accept the philosophy on which the reform of mathematics is based if they are to implement the new pedagogy it envisions. At least one of the teachers in the study treated the unit as a traditional textbook, assigning students tasks and using class time to tell them how to do the tasks. All teachers in the study found it difficult to move from the role of classroom authority to student guide. In their words, it was hard "…not to teach." Yet, all said they had changed their conception of teaching after their use of the unit.

Some teachers felt insecure at the beginning of the unit. Teachers who had viewed themselves as teaching statistics at the outset of the trial, began to view themselves as helping students learn how to use statistics to solve problems in the next few weeks. Suddenly there were no right answers; instead, there were several possible solutions to problems. To help students learn to solve problems, teachers learned to probe students about their understanding to listen to what students had to say, and finally to interpret students' explanations; communication became an essential part of the classroom enterprise.

Students had little difficulty adapting to the new materials and new ways of learning mathematics. While all students indicated they enjoyed the unit on data visualization, some did not view it as "real" mathematics. Students who flourished in mathematics classes that emphasized practice and replication found themselves challenged in new ways when working through the unit. Some who had been considered "poor in math" were able to succeed and some who had been considered "good in math" suddenly found themselves less successful.

Earlier assessment research by the Dutch had suggested that they correlations among scores for written restricted-time tests, oral tests, and take-home tests were low. Different assessment strategies, they concluded, were measuring different capabilities. Whitnall teachers also found that students who were best at traditional tests were not always best at the essay or similar product-development tests; different forms of assessment presented more equitable views of all students' achievement.

Students did experience some frustration as their teacher's behavior changed. One student remarked, "How can I do something if you don't tell me what to do?" Students who were used to taking short cuts to avoid reading questions found that key words were less useful, and students who were used to answering yes or no questions found they were now forced to justify their conclusions. The attitudes of students changed during the five-week trial: they increased their appreciation of common sense, discussion, and creativity, and they developed the view that mathematics is more than the memorization of rules.

Both the level of student engagement and the quality of their learning activity increased during the study. Observers noted that students engaged in "animated and purposeful discussions about its activities." One teacher remarked that "most of the time, in most of the classes, all the students were engaged in doing mathematics." While teachers reported that changing their teaching approach was a genuine struggle, they were eager to try a second unit. They also concluded the project had value—in their words (de Lange, van Reeuwijk, Burrill, & Romberg, in press): The students were excited about ideas—they were thinking and interpreting problems that were real and not contrived. No one said, "When will I ever need this?" They were learning to listen to each other as some raised valid points others had not considered…The creativity and ingenuity of many of the students was exciting. All of the students had the opportunity to succeed, and, in doing so exceeded our expectations. Consistently there was evidence of higher-order thinking and analysis in all of the classes, not just in the honors class. (p. 194-195)

The authors of the study conclude that, based on their experiences while carrying out the study, there is a danger that schools, teachers, educators, and publishers will make only "nominal" changes in their practices and then consider themselves in line with the NCTM Standards (1989). There is a passive view—held by teachers, their students, and the public—that equates teaching with telling students what to learn and how to learn it rather than with guiding students' learning process. Changes in instructional practices, according to this study, must accompany changes in materials.

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