Designing Middle-School Mathematics Materials Using Problems Set in Context to Help Students Progress From Informal to Formal Mathematical Reasoning

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February 2001*

* See “About this chapter,” p. i.

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ABOUT THIS CHAPTER

An earlier version of this paper appeared in *Mathematics in the Middle* (1998), published by the National Council of Teachers of Mathematics.

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SUPPORT

This manuscript and the research described herein are supported by the National Science Foundation (Grant #ESI-9054928) and the Educational Research and Development Centers Program (PR/Award Number R305A600007-01), as administered by the Office of Educational Research and Improvement, U.S. Department of Education, and by the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison. The opinions, findings, and conclusions do not necessarily reflect the views of the supporting agencies.

The preparation of this manuscript was supported, in part, by the National Science Foundation (Grant No. REC-9554193) and by the National Center for Improving Student Learning and Achievement in Mathematics and Science under the auspices of the Department of Education's Office of Educational Research and Improvement. Any opinions, findings, or conclusions are those of the authors and do not necessarily reflect the views of the supporting agencies.

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TABLE OF CONTENTS

Designing Middle-School Mathematics Materials Using Problems Set In Context To Help Students Progress From Informal To Formal Mathematical Reasoning

Designing curricular materials that emphasize students' understanding of the mathematics they are learning, and supporting implementation of those materials in classrooms, is not easy. This chapter has been written to illustrate how one group approached that task during the past decade.

The engineering task faced in this project involved creating a learning sequence of activities within each of the mathematical domains of middle-school science (i.e., algebra, number, geometry, statistics/probability), with each activity justifiable in terms of potential end points in the learning sequence. This approach to designing activities involved selecting contexts that could be mathematized, an approach Hans Freudenthal (1987), in describing the approach used in reforming Dutch school mathematics curriculum, termed Realistic Mathematics Education (RME). The sequencing of these activities required making fundamental design decisions about the intended pathways toward developing students' mathematical understanding. In these sequences, the initial contexts, similar to those used in the RME approach (Cobb, 1994), function as paradigm cases; the instruction includes activities that lead to students' modeling of their informal mathematics using simple notations; and students' models, with appropriate guidance from teachers, evolve into models for increasingly abstract mathematical reasoning.

In the pages that follow, we present an overview of the project, examine examples from the algebra strand of the curriculum to illustrate how problems in context provide students an opportunity to learn the interconnected ideas in that domain, present evidence of the quality of the curriculum, and conclude by discussing issues related to the implementation of these materials in classrooms, in light of the findings of the research conducted during the developmental process.

Overview of the Mathematics in Context *Project*

The project described here, "The Development of an 'Achieved' Curriculum for Middle-School Mathematics," was funded in 1991 by National Science Foundation. The resulting curriculum, *Mathematics in Context* (MiC; National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997–1998) reflects the content, teaching, and assessment standards for school mathematics proposed by the National Council of Teachers of Mathematics (1989, 1991, 1995). Collaborating on this project were research and development teams from the National Center for Research in Mathematical Sciences Education (NCRSME) at the University of Wisconsin–Madison, the Freudenthal Institute (FI) at the University of Utrecht (The Netherlands), and a group of field-test teachers in the United States. MiC materials consist of 40 curriculum units (10 at each grade level, Grades 5–8) and assessment materials; a teacher's guide for each unit; and two supplementary packets, News in Numbers (van den Heuvel-Panhuizen, Shafer, Foster, & Simon, 1997), which provides extra opportunities for students to develop estimation skills in contexts similar to those in the curriculum units, and Number Tools I and II (Burrill & Cole, 1997–1998), which provide worksheets focusing on students' enhancement of basic mathematics skills.

MiC was designed to support teaching as envisioned in the NCTM *Professional Standards for Teaching Mathematics* (1991). Each unit includes tasks and questions designed to engage students in mathematical thinking and discourse, with some activities designed to extend students' strategies and ideas to new problem situations. Students are expected to explore mathematical relationships, develop their own strategies for solving problems, use appropriate problem-solving tools, work together cooperatively, and value each other's strategies. They are encouraged to explain their thinking as well as their solutions. Teachers are expected to help students develop common understanding and usage of the terms, signs, symbols, and rules of mathematics in order to assist students in articulating their thinking.

PHILOSOPHY OF THE CURRICULUM

The MiC curriculum is built on the mathematical philosophy expressed by the mathematician William Thurston (1990): "Mathematics isn't a palm tree, with a single long straight trunk covered with scratchy formulas. It's a banyan tree, with many interconnected trunks and branches—a banyan tree that has grown to the size of a forest, inviting us to climb and explore" (p. 8). This curriculum emphasizes making connections among mathematical topics and domains and making connections between mathematics and real-world problem solving. Its roots are grounded in quantitative and spatial situations, and its trunks, referred to in MiC as strands, are *number* (whole numbers, common fractions, ratio, decimal fractions, percents, and integers), *algebra* (creation of expressions, tables, graphs, and formulas from patterns and functions), *geometry* (measurement, spatial visualization, synthetic geometry, coordinate and transformational geometry), and *statistics and probability* (data visualization, chance, distribution and variability, and quantification of expectations). Over the course of this 4-year curriculum, middle-school students explore and connect the mathematical strands. Although each unit in MiC emphasizes specific topics within a particular strand, most units involve ideas from several and emphasize the interconnectedness of those ideas.

This instructional design and sequencing of the MiC units was based on two related assumptions. The first was that students come to understand mathematics concepts from their experiences solving problems. Instruction in MiC starts with contexts demanding mathematical organization (contexts which can be "mathematized"). Freudenthal (1987) argued that instruction should begin with activities that contribute to mathematization and that, when students learn mathematics divorced from experiential reality, it is quickly forgotten, or they are unable to apply it. For students, making sense of a situation by seeing and extracting the mathematics embedded within it involves learning to represent quantitative and spatial relationships in a broad range of situations; to express those relations using the terms, signs, and symbols of mathematics; to use procedures with those signs and symbols, following understood rules, to carry out numerical and symbolic calculations; and to make predictions and interpret results based on the use of those procedures. This approach assumes students need to understand the rationale for the use of the mathematical terms, signs, symbols, and rules—"notions" that humans have invented and the meanings of which, over time, they have agreed on. Additionally, these notions have proven useful for particular purposes ranging from solving problems in engineering and science, to understanding the human condition, to creating new and abstract mathematics.

The second assumption was that the sequence of contextual activities should be designed to help students gradually develop methods for modeling and symbolizing problem situations. Because all activities are justifiable in terms of end goals in a learning sequence and perceived as a means of helping students progress from informal to formal semiotics, students' informal models, with teacher support and guidance, develop into models for increasingly abstract mathematical reasoning (Gravemeijer, 1991). The implication is that rather than starting with the presentation of formal terms, signs, symbols, and rules and expecting students to use these to solve problems (too commonly done in mathematics classes), activities should lead students to the need for the formal semiotics of mathematics. This development of ways of symbolizing problem situations and the transition from informal to formal semiotics ("progressive formalization") are important aspects of MiC.

The Development Process

The development of the MiC materials took six years. Initially, an international advisory committee of mathematics educators, mathematicians, scientists, curriculum supervisors, principals, and teachers was formed to ensure that MiC conformed to the goals and philosophy of the NCTM Standards (1989, 1991). This committee met and prepared a blueprint document to guide the development of the materials.

Freudenthal Institute staff then prepared initial drafts of individual curriculum units based on the blueprint, which researchers at the University of Wisconsin–Madison modified and developed further to create a curriculum appropriate for U.S. students and teachers. This joint creation of the MiC materials was not always easy. The Dutch researchers emphasized mental calculations and number sense rather than mastery of procedural routines and expected students to reflect on answers, strategies, and procedures and to judge the sensibility of their work. They were uneasy with or had concerns about paper-and-pencil computational skills. All groups struggled with the equity and authenticity of many of the instructional activities.

Pilot versions of the individual units were tested in middle-school classrooms in Wisconsin, and both students and teachers provided feedback and suggestions for revisions. Some activities and even one entire unit, which provided excellent opportunities for mathematization, were finally discarded or drastically modified when objections were raised, during review or during classroom trials, about the authenticity or suitability of a context.

The units were then revised and field-tested at additional schools in other states and in Puerto Rico. The field tests included trials of the entire year's curriculum at each grade level. Data collected during the field tests were used to revise the units again and to prepare detailed guides for teachers, the final step before commercial publication of the materials in both English and Spanish by Encyclopaedia Britannica.

An Illustration of the MiC Approach: The Algebra Strand

THEMES

Three themes run through the 14 units associated with the algebra strand: the study of change, the consideration of constraints, and the study of patterns. Much as themes in a symphony, sometimes one theme is stronger than the others; sometimes themes are evident simultaneously. Overall, the themes set the tone and determine the shape of the algebra strand.

Students study *change* in water levels in the fifth-grade unit Dry and Wet Numbers (Streefland, Roodhardt, Cole, & Brinker, 1997) as the basis for developing notation and ways of computing with positive and negative numbers. In the sixth grade unit Tracking Graphs (de Jong, Querelle, Meyer, & Simon, 1998), students produce their first graphs by analyzing phenomena that change over time (e.g., temperature). Studying change plays a central role in the seventh-grade unit Ups and Downs (Abels, de Jong, Meyer, Shew, Burrill, & Simon, 1998) as students graph and describe different kinds of change (e.g., linear, exponential, periodic). It is also central in one of the culminating eighth-grade units, Growth (Roodhardt, Burrill, Spence, & Christiansen, 1998). The study of change is important elsewhere as students build knowledge of concepts such as slope or as they move to formal notation and procedures for things such as developing formulas or solving equations.

Considering *constraints* first becomes prominent in the seventh-grade unit Decision Making (Roodhardt, Middleton, Burrill, & Simon, 1998) as students solve a realistic problem about building housing on a reclaimed landfill. Later, in the eighth-grade unit Graphing Equations (Kindt, Wijers, Spence, Brinker, Pligge, Simon, & Burrill, 1998), students develop the understanding and skills needed to graph linear inequalities and learn to use these graphs to define restricted regions on the coordinate system. Considering constraints is again prominent in the eighth-grade unit Get the Most Out of It (Roodhardt, Kindt, Pligge, & Simon, 1998) as students tackle a complicated problem in which they use all of their graphing skills and accumulated knowledge of linear programming.

Students first study *patterns* in the fifth-grade unit Patterns and Symbols (Roodhardt, Kindt, Burrill, Spence, 1997) as they begin using symbols to represent patterns they see in such things as growth rings of a tree or in blocks used to make a border along a garden. Recognizing and describing patterns is important as students write algebraic formulas in the sixth-grade unit Expressions and Formulas (Gravemeijer, Roodhardt, Wijers, Cole, & Burrill, 1998) and in the seventh-grade unit Building Formulas (Wijers, Roodhardt, van Reeuwijk, Burrill, Cole, & Pligge, 1998). The eighth-grade unit Patterns and Figures (Kindt, Roodhardt, Simon, Spence, & Pligge, 1998) centers on the study of patterns as students analyze sequences and write formulas for finding terms in the sequence. In this unit, they encounter several classic topics in mathematics that deal with patterns (e.g., Pascal's Triangle). Patterns also play an important part as students write formulas and equations, work on graphing, describe various kinds of change, and make connections between equations and graphs.

Progressive Formalization of Concepts

Throughout the MiC curriculum, students acquire mathematics concepts and skills through representing and analyzing real and realistic situations, and solving problems related to those situations. In the 14 units associated with the algebra strand (see Figure 1), the progressive formalization of the mathematics involves, first, having students approach problems and acquire algebraic concepts and skills in an informal way. They use words, pictures, and/or diagrams of their own invention to describe mathematical situations, organize their own knowledge and work, solve problems, and explain their strategies. In later units, students gradually begin to use symbols to describe situations, organize their mathematical work, or express their strategies. At this level, students devise their own symbols or learn certain nonconventional notation (e.g., arrow language). Their representations of problem situations and explanations of their work are a mixture of words and symbols.

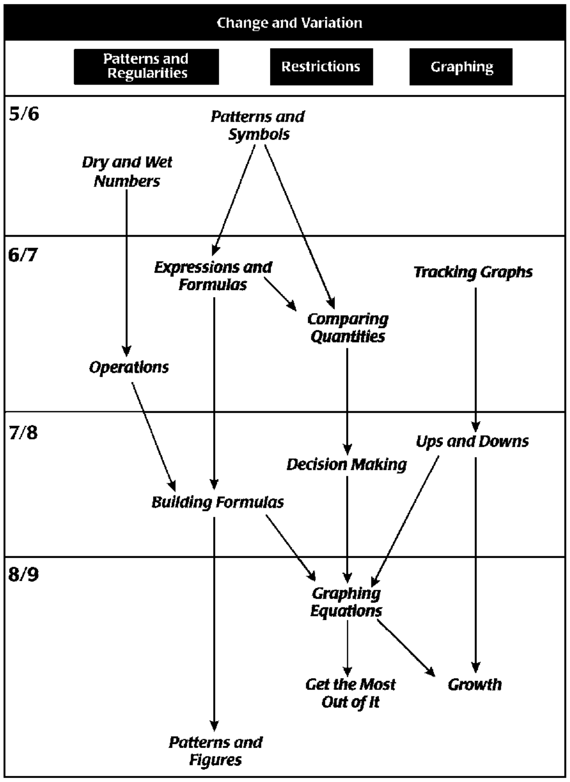

Figure 1. A map of the MiC strand.

In the later MiC units, students learn and use standard conventional algebraic notation for writing expressions and equations, for manipulating algebraic expressions and solving equations, and for graphing equations. Movement along this continuum is not necessarily smooth nor all in one direction. Students move back and forth among levels of formality depending on the problem situation or on the mathematics involved: Although they actually do algebra less formally in the earlier grades, they are not forced to generalize their knowledge to a more formal level, nor to operate at a more formal level, before they have had sufficient experience with the underlying concepts.

As a consequence of studying the problems in this sequence of units, students have the opportunity to reason, reflect on, and use the terms, symbols, and rules of algebra.

DISCUSSION OF EXAMPLE UNITS

To illustrate the kinds of problem situations students are expected to investigate in MiC and the way these situations have been orchestrated in the algebra strand, the contents of five related units are presented in the sections that follow.

Patterns and Symbols. The fifth-grade unit Patterns and Symbols (Roodhardt, Kindt, Burrill, & Spence, 1997) introduces students gradually to the use of symbols as an efficient way to represent real situations. Throughout the unit, students observe patterns in realistic situations and use symbols to describe them. In one lesson, for example, students are given information about the way the red and black growth rings on a snake develop. They describe this growth pattern symbolically, systematically record the number of red and black rings for a snake's first five growth cycles, then attempt either to predict the number of red and black rings a snake would have after a certain number of growth cycles or to determine the age of a snake with a given number of red and black rings. As students progress through the unit, they are introduced to concepts of equivalence and opposites (or inverse relationships), learn to find symbolic ways to represent a certain number of repetitions of a pattern, and, later in the unit, begin to substitute numbers for symbols as they try to figure out, for example, how many blocks (placed in a given pattern) will be needed to make a wall of a given length.

Expressions and Formulas. Many problems in the early sixth-grade unit Expressions and Formulas (Gravemeijer et al., 1998) are posed in the context of buying and selling. Among other things, students make change; determine grocery bills when buying produce, meat, or cheese by the pound; and figure out what a plumber's bill will be. In order to keep track of the sequences of calculations necessary to do these problems, students are introduced to several new tools. Given, for example, a story about someone earning and spending various amounts of money, students might learn to use the following arrow language to help them figure out how much money was left:

	+10		–2		+5		+5		–2.75		–3	
37	⟶	47	⟶	45	⟶	50	⟶	55	⟶	52.25	⟶	49.25

$49.25 is left

As problems and calculations become more complicated (e.g., buying three kinds of produce, each at a different price per pound), students adapt their arrow language to include multiplication and division and add another calculation tool, tables. When dealing with problems requiring multiplication/division *and* addition/subtraction, students learn about the order of operations and use another new tool, arithmetic trees, to help them organize and keep track of their work. Finally, students begin to generalize their calculations for specific problems to represent broader mathematical relationships. For example, for a problem about buying produce at varying prices per pound, students might form a generalization such as —

weight of tomatoes X 1.50 + weight of grapes X 1.70 + weight of green beans X 0.90 = total cost

or create an arithmetic tree such as the one shown in Figure 2.

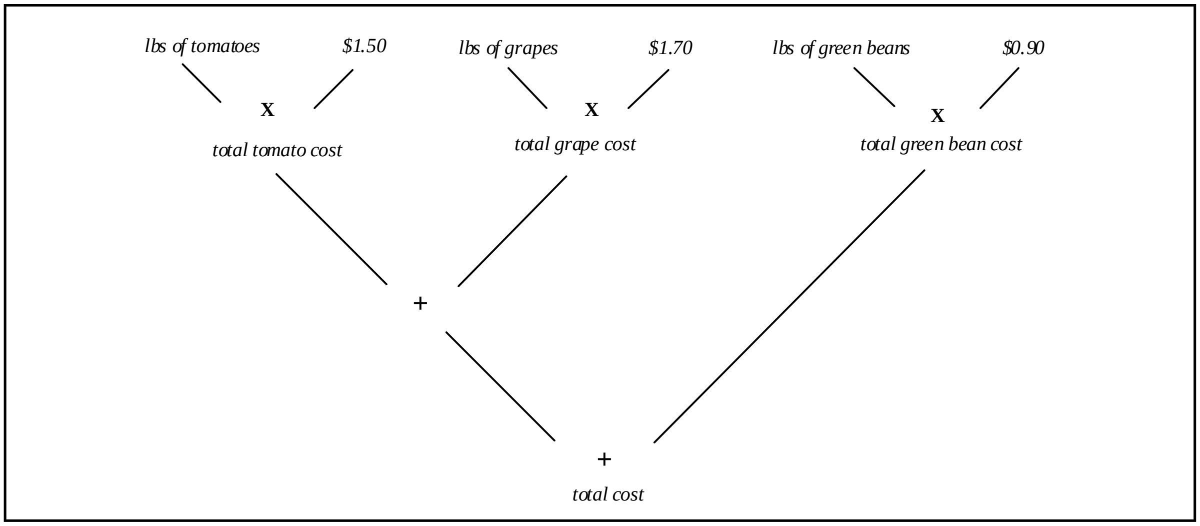

FIGURE 2. AN EXAMPLE OF AN ARITHMETIC TREE.

Operations. In the late sixth-grade unit Operations (Abels, Wijers, Burrill, Simon, & Cole, 1998), students examine patterns on a grid such as the one in Figure 3. First they use arrows, then (+) and (–) symbols, to describe moves on a grid that produce patterns (some of their own making). Students continually formalize what they are doing until they are working on a Cartesian coordinate system using ordered pairs.

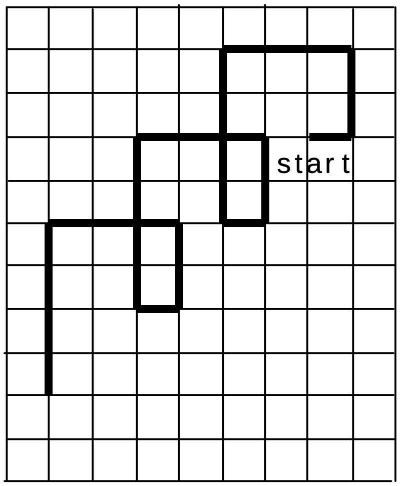

FIGURE 3. AN EXAMPLE OF THE TYPE OF GRID PATTERNS STUDENTS CREATE AND/OR DESCRIBE FOLLOWING INSTRUCTIONS USING ARROWS AND (+) AND (–) SIGNS.

In other sections of this unit, students solve problems involving making calls across time zones, calculating temperature changes or altitude changes on a mountain climbing trip, and playing a game that requires adding and subtracting positive and negative numbers. They use chips marked (+) or (–) as tools to aid their calculations and solve realistic problems such as the following:

> *For 10 days, high temperatures were recorded. On four days it was –3°C; on three days it was –2°C; on one day it was –1°C; and on two days it was +2°C.*
>
> *a. Explain why you can begin to find the mean temperature as follows:*
>
> *(4 x –3) + (3 x –2) + (1 x –1) + (2 x 2) =*
>
> *b. Finish the calculation for the mean temperature.*
>
> (Abels, Wijers, et al., 1998, p. 40)

One possible student solution for this problem might be the following:

> a. Because there were four –3s, three –2s, one –1, and two +2s, you should multiply the temperature times the number of times that temperature occurred and add it all up. Calculating gives the total for the 10 days.
>
> b. 4 x –3 + 3 x – 2 + 1 x –1 + 2 x 2
>
> = –12 + –6 + –1 + 4
>
> = –15
>
> The mean is –15 ÷ 10 = –1.5°C.

By the end of the unit, students have formalized notation and procedures for adding, subtracting, multiplying, and dividing with positive and negative numbers.

Building Formulas. In the early seventh-grade unit Building Formulas (Wijers et al., 1998), students use formulas to solve a variety of realistic problems. Problem situations involve, among other things, designing and mathematically describing patterns for tile walkways or for a metal framework used in building construction, converting temperatures from Fahrenheit to Celsius, figuring heart rate as related to age and exercise, and designing staircases to meet various building code requirements for rise (height of a step) and tread (depth of step).

Students encounter the distributive property informally as they describe brick patterns for a garden border. They then formalize both the concept and the notation, recognizing and using equivalences such as 4 (2S + 5L) = 8S + 20L. Students also encounter the need to square and “unsquare” numbers and use a variation of their familiar arrow language to express this. They then learn formal notation and use calculators to find square roots more exactly. In one problem, for example, students are told that a giant fungus discovered in northern Michigan in 1992 covered an area of 154,000 square meters. They imagine the fungus is in the shape of a square and are asked to find, to the nearest whole number, the length of the side of this fungus square and to explain their strategies.

Patterns and Figures. In the eighth-grade unit Patterns and Figures (Kindt, Roodhardt, et al., 1998), students start by studying sequences (arithmetic progressions) in which any number in the sequence (any term) can be found by adding or subtracting a certain number from the previous number in the sequence. Students attempt to describe the recursive rule governing the sequence in words and algebraic formulas. They then also attempt to develop a direct rule or formula describing how to find any term in the sequence without knowing the previous term. Students apply and practice this in the context of real situations (e.g., the arrangement of seats in a Greek amphitheater, flying patterns of migrating birds).

Students learn they can combine sequences (by addition and subtraction), encountering or revisiting other important mathematical topics such as binary coding, tessellations, or triangular numbers. This unit broadens their mathematical experience and makes connections between algebra and geometry. Among other things, they determine and describe the relationship among the vertices, edges, and faces of pyramids and relate this to their work with the vertices, edges, and faces of prisms in the seventh-grade geometry unit Packages and Polygons (Kindt, Spence, Brinker, & Burrill, 1998), and they formally recognize Euler's formula. Finally, through studying sequences that involve squaring numbers, students are formally introduced to Pascal's triangle.

Evidence on the Quality and Effectiveness of the Materials

Over the six years in which MiC materials were developed, considerable information was gathered about the quality and effectiveness of the materials (cf. Romberg, 1997; Romberg & Pedro, 1997). During both the pilot and field tests of the materials, information was gathered from teachers, students, administrators, and parents through surveys, teacher logs, unit tests, and classroom observations. Most of the data were used to revise and improve the activities and the teacher's guides that accompany the units. The evidence about the program is briefly summarized here.

Dutch Evidence

MiC had its roots in the Dutch approach to mathematics. In 1992, Dutch educators compared on 29 mathematics scales the achievement of students who used realistic textbooks with that of students who used traditional materials (Bokhove, 1995). Students who used realistic textbooks scored higher than those who used traditional textbooks on 19 of the 29 scales (12 were statically significant at the .05 level). On only one scale did students using traditional texts do better than students using realistic texts. Although there are considerable differences among the Dutch realistic texts and MiC and the traditional texts in both countries, this evidence provided support for initiating the development of a realistic curriculum for students in this country.

REVIEWS OF CONTENT

Four University of Wisconsin–Madison faculty members with expertise in number, algebra, geometry, or statistics/probability were asked to review the content of the student units for a given strand. Dr. Simon Hellerstein, a professor of mathematics and former chair of the department of mathematics, in a review of the units in the algebra strand, wrote—

> Incorporating pre-algebraic ideas and skills into the middle-school math curriculum, culminating in the eighth grade with some basic formal algebra, is a splendid undertaking. The end goals of the algebra strand as listed in teacher guide are commendable. In those schools where the goals are actually achieved, students will have been well served and better prepared to tackle high school algebra. . . . The units seem likely to engage most students' interest. Most of the "real world" phenomena and activities used to introduce and illustrate the mathematics seem well chosen to motivate the ideas. This approach in itself is an enormous improvement over the dull traditional middle-school mathematics learning experience. Middle-school math has been a "killing ground" for interest and curiosity about mathematics. Where the students are encouraged to find their own strategies or to write a description of their reasoning, the role of the teacher, while important for the rest of the material, becomes critical. I am concerned about the number of teachers who have sufficient comprehension to recognize good ideas and to help flesh them out or to be able to be critical, as well as kind and remediating, when reviewing such student efforts. Encouraging independent thinking is ideal; reinforcing sloppy independent thinking is poor. Controlling the latter while not at the same time also squelching the former is a challenge that too few teachers may be able to handle. (Personal communication, March 30, 1995)

Similar comments about the quality of the mathematics in each strand and the instructional approach used in the curriculum were made by the other three faculty reviewers. At the same time, however, each cautioned developers about the difficulties he could see in implementing the program in many schools.

INFORMATION COLLECTED FROM TEACHERS

On every unit they taught, teachers were surveyed about the forms of assessment they used, the amount of time allowed for the assessment activities at the end of the unit, and the results (for specifics, see Romberg, 1997). For example, one teacher of the field-tested version of Expressions and Formulas said, "I was very pleased with the [students'] understanding of the mathematics in the unit. Students understood the concepts and were able to apply them on the assessment [activity]."

Teachers were also asked to provide one or more copies of student work; examples of low, medium, and high student performances; and a list of scores for the end-of-unit assessment activity, with students identified by gender and grade level. The comments on the field-test version of Expressions and Formulas were both positive and negative (see Table 1).

One significant outcome for teachers in these studies was a changed view of their students and their students' capabilities. In one field-test summary, we noted that "most teachers were amazed that students could communicate and were excited about talking and doing math. They were able to make connections to the world around them and to other areas of study in school" (Romberg & Shafer, 1995, p. 16). Burrill (quoted in de Lange et al., 1993) noted that, "The students were excited about ideas — they were thinking and interpreting problems that were real and not contrived. No one said, 'When will I ever need this?'" (p. 158).

TABLE 1. TEACHER COMMENTS ON EXPRESSIONS AND FORMULAS

General Comments	
"Students as a whole were quite successful as exhibited by their discussions, journals, and assessments."	"Formulas were generally hard for students."
"Content was presented in such a way that it was meaningful and related to a variety of situations in everyday life."	"There were no parts my students could accomplish in small groups."
"Arrow language was very useful, especially in helping students reverse the order when looking for a factor rather than a final product or answer."	"This unit needed teacher directions."

Comments on Students' Understanding of the Content	
"Most students did well on all but the section about the Forest Fire Fighting, especially problems 3 and 4 where they had to think."	"Occasionally the students would not understand how a problem was worded, and we would have to discuss it as a group."
"When given a Continental Math League test, many students used reverse arrow strings to figure out answers!"	"Students struggled with every part of this unit except arrow language."
"I noticed several using trees to solve other math problems after we completed the unit."	"Students were confused and anxious but demonstrated effort to understand. Towards the end they 'clicked.' "
"Students were able to apply the concepts on the assessment."	

STANDARDIZED TESTS

Three school districts using the MiC curriculum for at least two years shared some district-collected data. The site coordinator for the field test in Ames, IA, reported seventh-grade student scores on the 1995 Iowa Aptitude Test and compared them with scores of seventh-grade students taking the test in 1994 (Delagardelle, 1995). She reported a 9% increase in the number of students earning scores in the top category on the test and a 7% decrease in the number of students falling into the lowest category on the test.

A Dade County, FL, administrator reported that in 1995 students in Grades 5, 6, 7, and 8, "significantly improved their test scores [on the Mathematics Applications Subtest of the Stanford Achievement Test] compared to the 1994 administration of the same subtest" (Cohen, 1995).

In the Puerto Rican field-test site, students studying MiC units at the Jesús Sanabria Cruz School were given the same standardized tests administered to all Puerto Rican students by the Puerto Rican Department of Education. In 1995, Jorge Lopez reported:

> Their regular track students participating in the MiC project obtained extremely high scores; in fact, except for 2 students of a group of 23, all students scored in excess of the 90th percentile for all of Puerto Rico. The remaining two students scored 82 and 84. The Title I students [students with academic deficiencies as of 1994] were segregated into two groups [for reasons having to do with the school's policies] and [those in] one of the groups were chosen to be participants of the MiC project. In the latter group, all students scored above the upper limiting score for the Title I program so that, in fact, all students were taken out of the Title I program. All of the students following the traditional curriculum remained as Title I students and did not show any improvement in their scores as compared to the previous year.

If reform curricula are to be widely used, it is critical that data be collected on student mathematical performance as consequence of the use of these curricula in classrooms. Because such data need to be related to the final, commercially published version of the curriculum, the project staff did not gather this type of evidence during the development and field testing of MiC. In 1996, however, the National Science Foundation funded a 4-year study, "A Longitudinal/Cross-Sectional Study of the Impact of Mathematics in Context on Student Mathematical Performance." This study is examining the impact of the final version of the MiC curriculum on the mathematical knowledge and understanding, attitude, and performance of middle-school students when used continuously over a 4-year period. The research study of MiC began in 1996. Results are expected in 2001.

SUMMARY

The prepublication evidence showed that *Mathematics in Context* is a quality product. Preliminary findings from the longitudinal study of its use also suggest that, if appropriately implemented in middle-school classrooms, the use of the *Mathematics in Context* curriculum will be an effective means of supporting not merely students' understanding and use of mathematics to solve nonroutine problems, but also students' development of the mathematical literacy needed in the 21st century.

Implementation Issues

From the beginning, we were confident that we would be able to develop a middle-school curriculum that met the expectations of the NCTM Standards, but anticipated that implementation would be difficult. From our research carried out during the pilot and field testing of the materials, we also found that *wide-scale* implementation of such a reform curriculum will be very hard. In every classroom in which MiC was used during the pilot and field testing of the materials, the traditional school culture and the common instructional routines for school mathematics were strongly challenged. This occurred in strong part because of the ways MiC differed from the traditional texts, but as Sarason (1971) argued, "any attempt to introduce a change into the school involves [challenging] some existing regularity, behavioral or programmatic" (p. 3). This element of implementing a reform curriculum will continue to be a major issue in school districts.

The case studies and data discussed in this section have been drawn from a series of ongoing case studies and the formative information gathered during field trials of the materials (Romberg, 1997) and focus on five areas: instructional sequence, coverage, content, authority and student capability.

INSTRUCTIONAL SEQUENCE

Initially, many teachers believed they would "gain a few new ideas that we could use in our classes once the project was over" (de Lange et al., 1993, p. 153). Instead, the approach changed the work environment for both teachers and students, from well-rehearsed routines to a variety of nonroutine activities.

In Weller's (1991) study of two traditional mathematics classrooms, he found a common daily pattern of instruction in both classes: "It was evident that a repeating pattern of instruction occurred which consisted of three distinctive segments: a review, presentation, and study/assistance period. This 'rhythm of instruction' was not unplanned or coincidental" (p. 128). All of the study teachers found that teaching an MiC unit constituted a departure from this traditional daily pattern. As Burrill (quoted in de Lange et al., 1993) reflected:

> The surprise came when we tried to teach the first lesson. There was little to "teach"; rather, the students had to read the map, read the keys, read the questions, determine what they were being asked to do, decide which piece of information from the map could be used to help them do this, and finally, decide what mathematics skills they needed, if any, in answering the question. There was no way the teacher could set the stage by demonstrating two examples (one of each kind), or by assigning five "seat work" problems and then turning students loose on their homework with a model firmly (for the moment) in place. (p. 154)

COVERAGE

The importance of "coverage" in traditional classrooms was characterized by Weller (1991):

> A goal of mathematics teachers was to cover a prescribed amount of material preparing students to enter the next level of mathematics study. The overall pace of instruction required the [teachers] to teach the textbook, cover-to-cover, as the mathematics curriculum required. The sequential order of concept presentation was determined by the textbook editor and thus embraced by the department. (p. 128)

Teachers using MiC often voiced concern over whether to teach traditional topics as the materials were being tested. The danger, of course, is twofold: the tendency, first, to augment reform instruction with traditional practices and, second, to modify and change activities so that they resembled past lessons. For example, B. Clarke (1995) found that, "there was also a tendency on the part of one of the teachers, having taught the lesson once, to vary the presentation in subsequent lessons, with increasing levels of structure, as a response to feelings of personal discomfort" (p. 159).

UNFAMILIAR CONTENT

B. Clarke (1995) found that "tasks within the unit and students' attempts at these led to the introduction of a variety of mathematical content areas, some of which were unfamiliar to the teachers" (p. 157). When this occurred, teachers were unsure how to proceed.

But even when the teachers were familiar with the mathematics, they were often unsure how to proceed because they had never taught that mathematics to students and generally did not well understand the learning process of their students. D. Clarke (1993) found that teachers floundered when "they lacked knowledge of most students' learning of the content of the unit and the means by which such knowledge could be obtained" (p. 222). This lack of familiarity with the specific mathematics in a unit and the potential connections to other content was often reflected in the assessment tasks teachers created. For example, van den Heuvel-Panhuizen (1995) found that "the teacher-made problems made clear that [they] had a preference for computational problems and problems which are firmly connected with the contexts and the tasks that are used in that teaching" (p. 70). Similar comments related to assessments were made by many of the field-test teachers when interviewed by the staff as a part of the review process (Romberg & Shafer, 1995).

AUTHORITY

Weller (1991) found that in traditional classrooms, "the expert knowledge of the teacher was deliberately subjugated to that of the textbook authors. As a result of that process, the teacher was able to camouflage his role as authoritarian, thus eliminating student challenges of authority" (p. 133). In contrast, students using MiC read, discussed, and made sense of tasks in their student booklets under the guidance of the teacher and with the help of their peers. This approach to instructional authority changed the work environment for both teachers and students.

In the field-testing of *Mathematics in Context,* teachers did not fall back on the authors (or the answer book) as they had when using traditional texts. When reflecting on the work of the two teachers in her study, B. Clarke (1995) commented that "the teachers encouraged conjecturing and inventing as students solved problems. This led to critical incidents for the teachers as they tried to understand the students' methods of solution on the spur of the moment. Insightful student responses led to more critical incidents than any other type" (p. 156). But she also found that for these teachers, "the reality of changing authority [was] difficult. There were a number of times when the students were able to provide the clarification of a difficulty. This was powerful, though potentially threatening to the authority of the teacher" (pp. 157–158). In most classes, teachers occasionally had to admit they did not know how to approach a problem and thus had to work on the tasks with the students as equals.

STUDENT CAPABILITY

One significant outcome was that teachers in these studies changed their perception of their students and their students' capabilities. In fact, all the teachers were surprised by the work their students were able to do. As one field-test teacher commented, "What we're finding in some of our work, too, is that the kids get things that we thought might be hard for them" (Romberg & Shafer, 1995, p. 8).

The impact of having students "do mathematics" in this manner had two consequences. As Burrill (quoted in de Lange et al., 1993) found, "Students who had been labeled as 'poor in math' found they could succeed, and some who considered themselves 'good in math' decided 'this isn't math' " (160). Van den Huevel-Panhuizen (1995) found similar results when teachers administered open assessment tasks:

> One teacher even discovered something new about the students' understanding by means of the test: "Some of my quieter students displayed a greater understanding than I had given them credit for, some displayed a sense of humor." (p.71)

This same teacher went on to make the following comment about the assessment for the unit: "I feel it offers more than most objective sorts of tests. It allows students to explain their thinking—a very valuable piece of information" (p. 71).

Summary

Information both from the case studies and from the field testing of *Mathematics in Context* make clear that this approach to mathematics instruction "represents, on the whole, a substantial departure from teachers' prior experience, established beliefs, and present practice. Indeed, they hold out an image of conditions of learning for children that their teachers have themselves rarely experienced" (Little, 1993, p. 130). External reviews and preliminary findings from the NSF-funded study of the curriculum provide strong evidence that the appropriate use of the curriculum in middle-school classrooms can substantially increase student achievement and understanding of mathematics.

As Chevelard (1988) pointed out, there is a gap between what is taught in school mathematics classrooms and what is learned there. He argued that this gap is created when we treat a pupil only as a "student" (someone who studies) and not as a "learner" (someone who learns). The approach taken in these classrooms by these teachers (who used *Mathematics in Context*) is intended to close this gap. For teachers, the transition from working with students to working with learners is difficult, but has an impact not merely on students' short-term mathematical achievement, but on their long-term mathematical understanding and literacy.

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