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CHANGING the teaching and learning of MATHEMATICS

During the past quarter of a century there has been considerable rhetoric about the need to reform the teaching and learning of mathematics. The 'standards-based reform movement' now under way in many countries, states and schools is the outcome in response to the calls for change. The question I am raising in this brief paper is — How would one know if the reforms have an influence on student learning? My response to this question involves clarifying four aspects of the question. First, the shift in epistemology about the learning of mathematics must be understood. Second, the systemic notions about schooling that follow from that shift need to be considered. Third, appropriate evidence related to the notions of schooling practices should be documented. And finally, if influence of the reform practices on student learning is to be determined, new means of assessment need to be developed.

Shift in epistemology

To illustrate this shift I have chosen to use the approach taken in the USA. In 1986 the Board of Directors of the National Council of Teachers of Mathematics (NCTM) established the Commission on Standards for School Mathematics to:

- Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields; and

- Create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision. (NCTM, 1989, p. 1).

The products of this charge were NCTM's three standards documents published in 1989, 1991, and 1995, and its recently published *Principles and standards for school mathematics* (2000).

The central tenet underlying this charge is for students to become mathematically literate. In James Gee's *Preamble to a literacy program* (1998) the term 'literacy' refers to the human use of language. In fact, one's ability to read, write, listen and speak a language is the most important tool we have through which human social activity is mediated. Each human language and each human use of language has both an intricate design and is tied in complex ways to a variety of functions. For a person to be literate in a language implies that the person knows many of the design resources of the language and is able to use those resources for several different social functions. Analogously, considering mathematics as a language implies that students not only must learn the concepts and procedures of mathematics (its design features), but they must learn to use such ideas to solve non-routine problems and learn to mathematise in a variety of situations (its social functions). The epistemological shift involves moving from judging student learning in terms of mastery of concepts and procedures to making judgements about student understanding of the concepts and procedures and their ability to mathematise problem situations. In the past, too little instructional emphasis was on understanding, and the tests used to judge learning failed to adequately provide evidence about understanding or ability to solve non-routine problems.

Reform schooling

A set of assumptions about instruction and schooling practices has been associated with this vision of mathematical literacy. First, all students can and must learn more and somewhat different mathematics than has been expected in the past in order to be productive citizens in tomorrow's world. In particular, all students need to have the opportunity to learn important mathematics regardless of socio-economic class, gender, and ethnicity. Second, we have long underestimated the capability of all students to learn mathematics. Third, some of the important notions we expect students to learn have changed due to changes in technology and new applications. Thus, at every stage in the design of instructional settings we must continually ask — Are these important ideas in mathematics for students to understand? Fourth, technological tools increasingly make it possible to create new, different and engaging instructional environments. Fifth, the critical learning of mathematics by students occurs as a consequence of building on prior knowledge via purposeful engagement in activities and by discourse with other students and teachers in classrooms.

This last assumption about the learning of mathematics is based on research, carried out in the last decade, showing that, in classrooms where the emphasis of instruction has shifted from mastery of facts and skills to understanding, students become motivated to learn and achievement at all levels has increased. Carpenter and Lehrer (1999) have characterised understanding in terms of five interrelated forms of mental activity, from which mathematical and scientific understanding emerges:

1. constructing relationships,
2. extending and applying mathematical and scientific knowledge,
3. reflecting about mathematical and scientific experiences,
4. articulating what one knows, and
5. making mathematical and scientific knowledge one's own.

Since all learning occurs as a consequence of experiences, and all humans have a variety of experiences, virtually all complex ideas in mathematics are understood by a student at a number of different levels in quite different ways. Furthermore, a student's level of understanding will change as a consequence of instructional experiences. Thus, the challenge is how to create classroom experiences so that a student's understanding grows over time. As recently stated in *How people learn*.

> Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classrooms (Donovan, Bransford & Pellegrino, 1999, p. 10).

Research in mathematics instruction has identified a series of steps that lead students to understanding. Students should begin their investigations with a problem that needs to be addressed that makes sense to them. The initial instructional activity should be experientially real to students so they are motivated to engage in personally meaningful mathematical work. This step involves raising questions about the problem situation. Hypothesis generation is a critical aspect of mathematical and scientific reasoning that rarely has been taught. Any instructional sequence assumes that each activity is justifiable in terms of some potential end points in a learning sequence. Paul Cobb (1994) states:

> This implies that students' initially informal mathematical activity should constitute a basis from which they can abstract and construct increasingly sophisticated mathematical conceptions. At the same time, the starting point situations should continue to function as paradigm cases that involve rich imagery and thus anchor students' increasingly abstract mathematical activity (p. 23-24).

Next, students need to identify information and procedures they could use to answer their questions. Cobb then goes on to argue that

> Instructional sequences should involve activities in which students create and elaborate symbolic models of their informal mathematical activity. This modelling activity might involve making drawings, diagrams, or tables, or it could involve developing informal notations or using conventional mathematical notations (p. 24).

Finally, one needs to build a coherent case about the quality of one's answer, developing an appreciation of standards of evidence and appropriate forms of argument.

The point is that, with appropriate guidance from teachers, a student's informal models can evolve into models for increasingly abstract mathematical and scientific reasoning. The development of ways of symbolising problem situations and the transition from informal to formal semiotics are important aspects of these instructional assumptions.

The complexity of instructional issues involved in creating classrooms that promote understanding include the interconnected roles of tasks on the one hand and how students and their teachers talk about mathematics on the other, how technological tools can help in the development of classrooms that promote understanding, the normative beliefs within a classroom about how one does mathematics, the organisational structures of the classroom, the role of professional development in helping teachers to develop their own classrooms that promote understanding, how the school, as an organisation, supports (or impedes) the work of teachers in developing and sustaining these classrooms, and how non-school agents (such as parents), agencies (districts), and their actions support (or impede) the development of these classrooms.

The fact is that the vision of reform should be focused on this non-routine pattern of instruction that allows students to become mathematically literate.

Appropriate evidence about changes in schooling practices

The problem with the vision of school mathematics, as outlined in the previous paragraphs, is that it involves ideals put forward by educational leaders, policy-makers, and professors about what mathematical content, pedagogy, and assessments should be. Implementation of such ideals can be undermined by a number of factors. For example, not all persons agree with the goal of mathematical literacy for all; some influential persons believe that the current course of study works reasonably well (particularly for their children), etc. In fact, as Labaree points out, during the past century calls for reform have had remarkably little effect on the character of teaching and learning in American classrooms (1999, p. 42).

In conventional classrooms the mathematical content is cut off from practical problem situations and taught in isolation from other subjects, students are differentiated by ability and sequenced by age, instruction is grounded in textbooks and delivered in a teacher-centred environment. Instead of changing conventional practices, the common response to calls for reform has been the 'nominal' adoption of the reform ideas. Schools have used the reform labels but did not follow most of the practices advocated. It is often a political necessity for schools and teachers to claim they are using a standards-based, reform program even if classroom practices have not changed (Romberg, 1985). Thus, to document the impact of any reform efforts in classrooms, one needs to examine the degree to which the reform vision actually has been implemented.

Being aware of this problem, the strategy which underlies the reform movement in the USA was based on the notion that, since we live in a supply-and-demand economy, if the mathematics community wanted different texts and tests, a demand would have to be created (Romberg, 1998). To respond to this challenge it was anticipated that the mathematical sciences community would follow an iterative strategy over several years. First, to create demand it was assumed that States, districts, and schools would arrive at consensus about the details of long-and short-range plans (with timetables) for change. Second, specific elements of the system to be targeted for change (curriculum materials, instructional methods, examinations, teachers, technology...) would be identified and prioritised. Third, demands would be made to textbook publishers, testing companies, staff developers, teacher education programs, etc. that they contribute the ingredients necessary for the desired changes in elements. Next, assuming that new materials and procedures consistent with the vision are supplied, they need to be tried out and feedback from this trial phase matched with the vision and the plan, and revisions made. Finally, the quality of student performance then should be judged in terms of what students are able to do, whether this meets the reform vision, and in turn whether this meets society's needs. Only by following such a strategy would real rather than nominal change actually occur.

Assessing mathematical literacy

To be consistent with the standards-based vision, the quality of student performance should be judged in terms of whether students are mathematically literate. This means that information should be gathered about what concepts and procedures students know with understanding, and how students can use such knowledge to mathematise a variety of non-routine problem situations. Only then can one judge whether student performance meets the reform vision, and in turn whether the changes meet society's needs.

Unfortunately the existing instruments commonly used to judge student performances in mathematics were not designed to assess mathematical literacy. Standardised tests used by school districts in the USA measure the number of correct answers a student can answer to questions about knowledge of facts, representing, recognising equivalents, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, manipulating expressions containing symbols and formulae in standard form, and doing calculations. Most state assessments, and the items included in the recent National Assessment of Educational Progress (NAEP), and in the recent Third International Mathematics and Science Study (TIMSS) are of this character. As such, at best they measure a student's knowledge of some of the 'design features' associated with mathematical literacy. Also, it is questionable as to whether such instruments measure understanding of such features. And none make any serious attempt to assess student capability to mathematise. Thus, to honestly assess the intended impact of standards-based reforms in mathematics, a new assessment system will need to be developed.

Fortunately there is one new international assessment framework emphasising literacy (reading, mathematical, and scientific) that has been designed for the Programme for International Student Assessment (PISA) by the Organisation for Economic Cooperation and Development (OECD, 1999). This program has been designed to monitor on a regular basis achievement of students as they approach the end of secondary school. Tests developed from this framework have just been administered in some 30 countries. It is premature to judge the quality of this program, but the framework they are using fits the reform epistemology. In particular, the notion of big ideas such as chance, change and growth, space and shape, etc., and the designation of three 'competency classes' defining the type of thinking skill needed are consistent with the reform rhetoric. The three competency classes are: (1) tasks requiring simple computations or definitions, (2) tasks requiring connections be made to solve straightforward problems, and (3) tasks requiring mathematical thinking, generalisation, and insight. Here students are required to engage in analysis, to identify the mathematical elements in a situation, and to pose their own problems.

Summary

The initial question raised in this paper — How would one know if the reforms have an influence on student learning? — now can be answered. One would know if:

- the shift in epistemology towards mathematical literacy is understood,
- a shift in schooling practices consistent with mathematical literacy can be documented, and
- assessments aligned with mathematical literacy have been developed and used to document student learning.

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Note

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